Video Signals

MORPHOLOGY

Mathematic Morphology

used to extract image components that are useful in the representation and description of region shape, such as

- boundaries extraction
- skeletons
- convex hull
- morphological filtering
- thinning
- pruning

Mathematic Morphology

mathematical framework used for:

pre-processing

- noise filtering, shape simplification, ...

enhancing object structure

- skeletonization, convex hull...

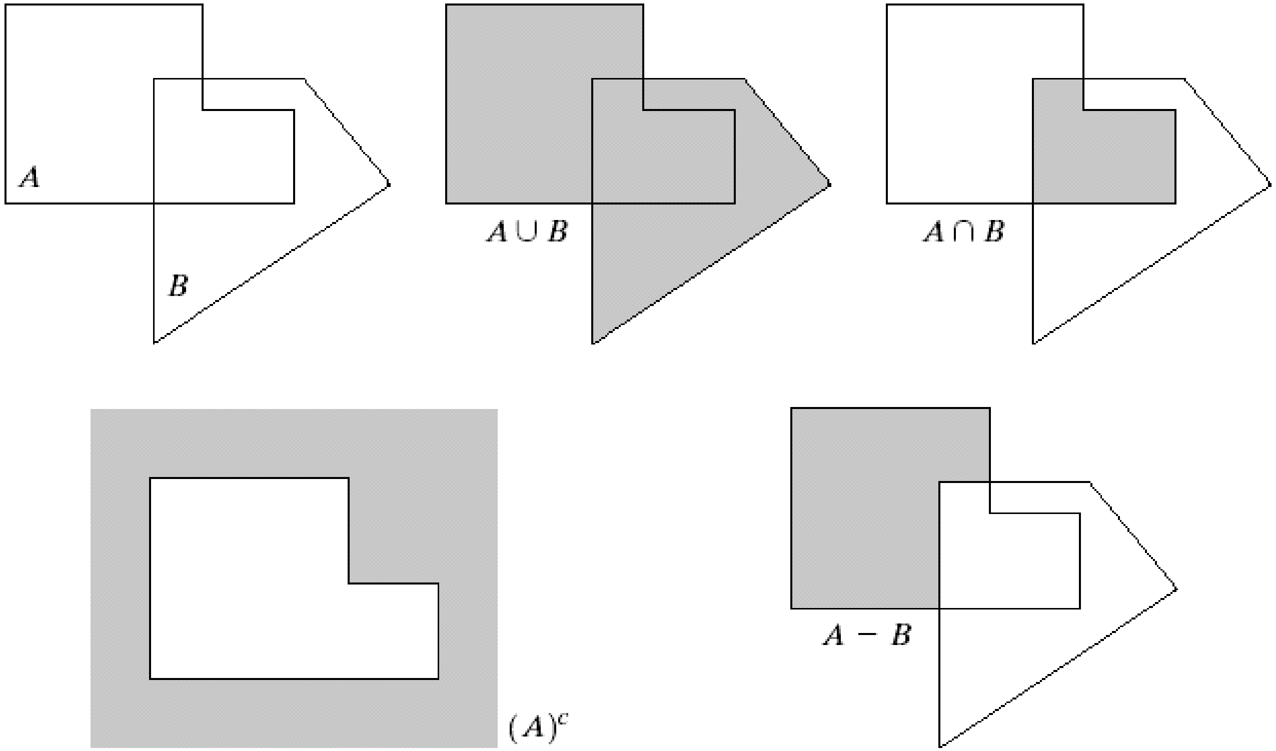
Segmentation

- watershed,...

quantitative description

- area, perimeter, ...

Basic Set Theory



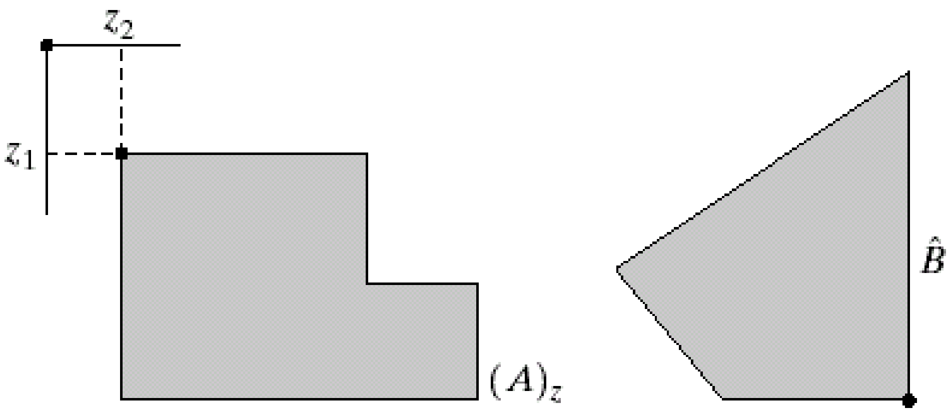
a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflection and Translation

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Logic Operations

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT (**p**) (also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Example

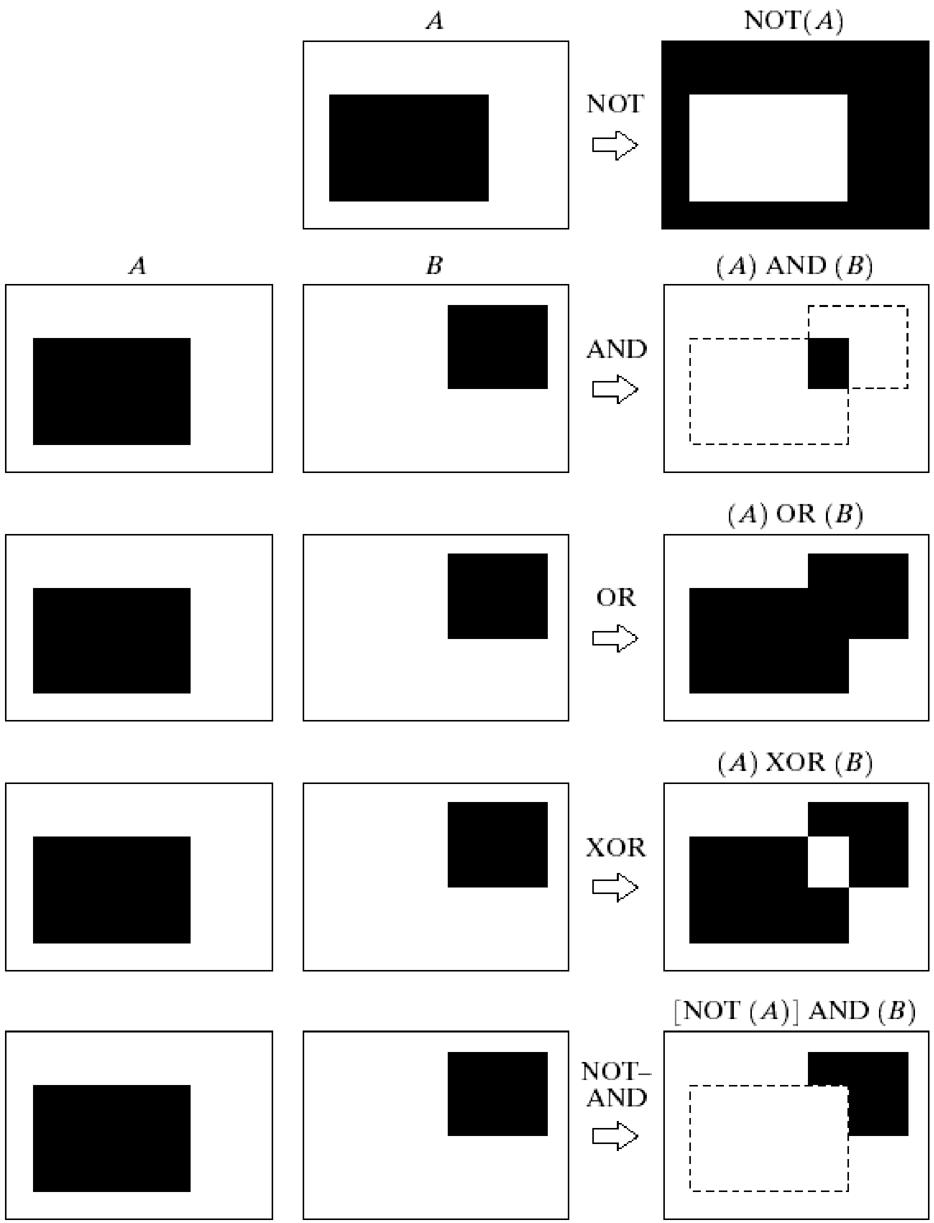
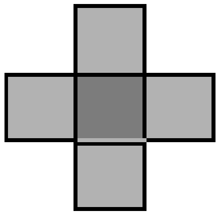


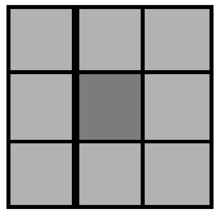
FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring element (SE)

- small set to probe the image under study
- for each SE, define origo
- shape and size must be adapted to geometric

properties for the objects

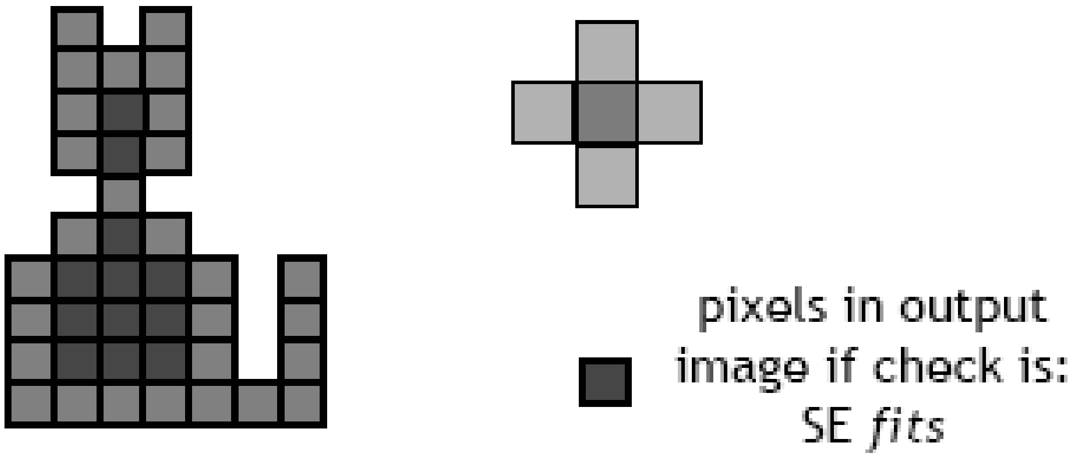




Basic idea

in parallel for each pixel in binary image:

- check if SE is ”satisfied”
- output pixel is set to 0 or 1 depending on used operation

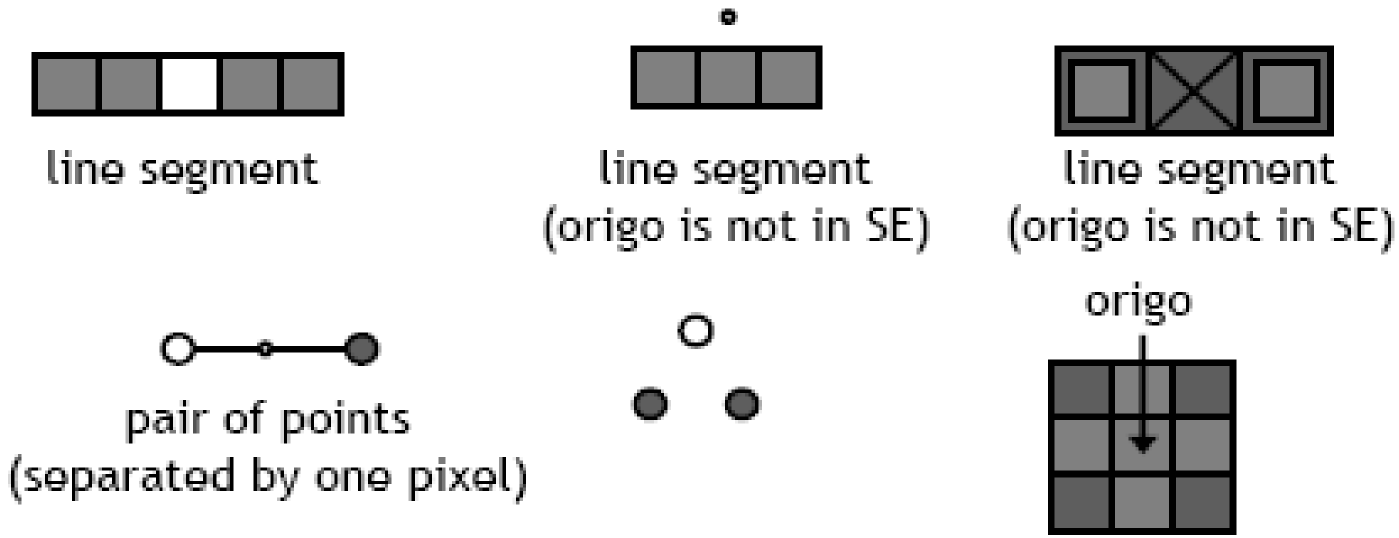


How to describe SE

many different ways!

information needed:

- position of origo for SE
- positions of elements belonging to SE



Basic morphological operations

Erosion

Dilation

combine to

keep general shape but smooth with respect to

- Opening → object
- Closing → background

Erosion

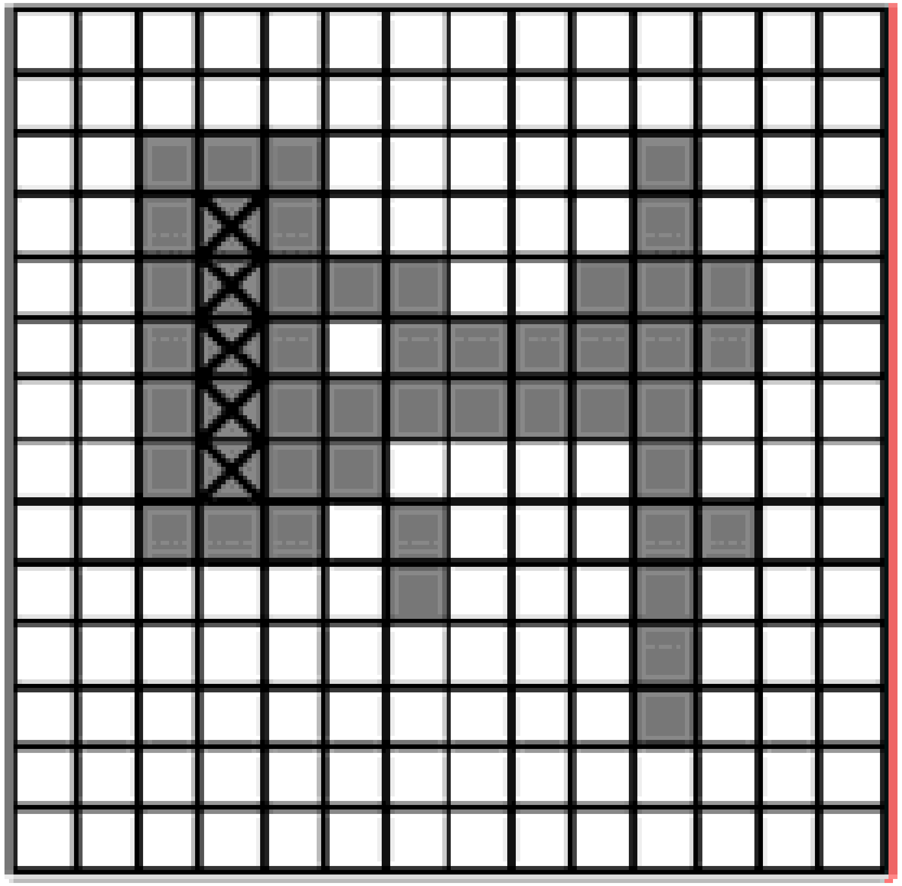
Does the structuring element **fit the set?**

erosion of a set A by structuring element B:
all z in A such that B is in A when origin of B=z

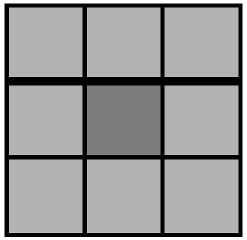
shrink the object

$$A \ominus B = \{ z | (B)_z \subseteq A \}$$

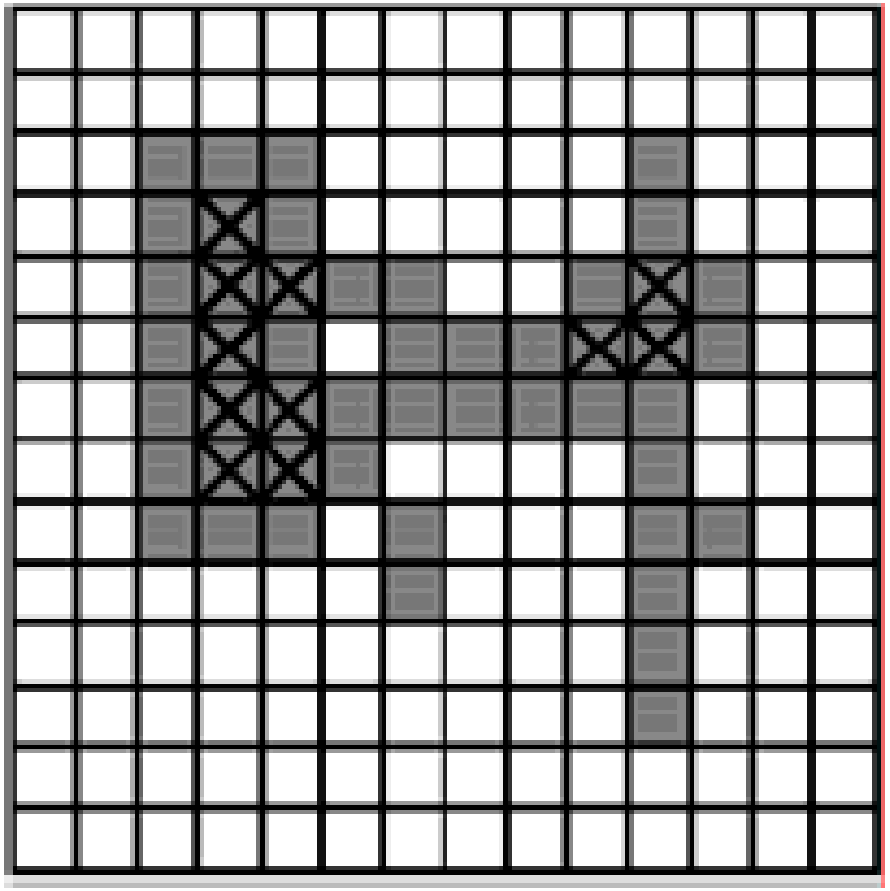
Erosion



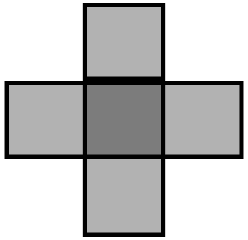
SE=



Erosion



SE=



Dilation

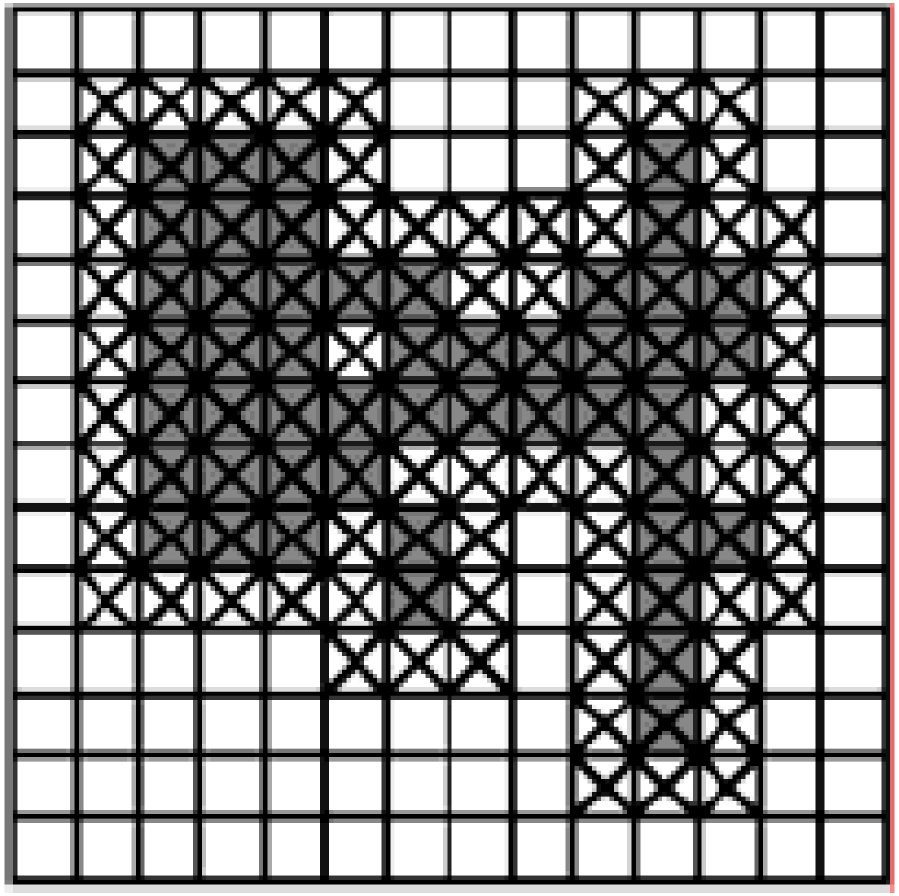
Does the structuring element **hit the set?**

dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z

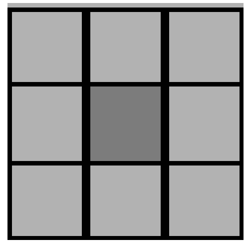
grow the object

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varnothing \}$$

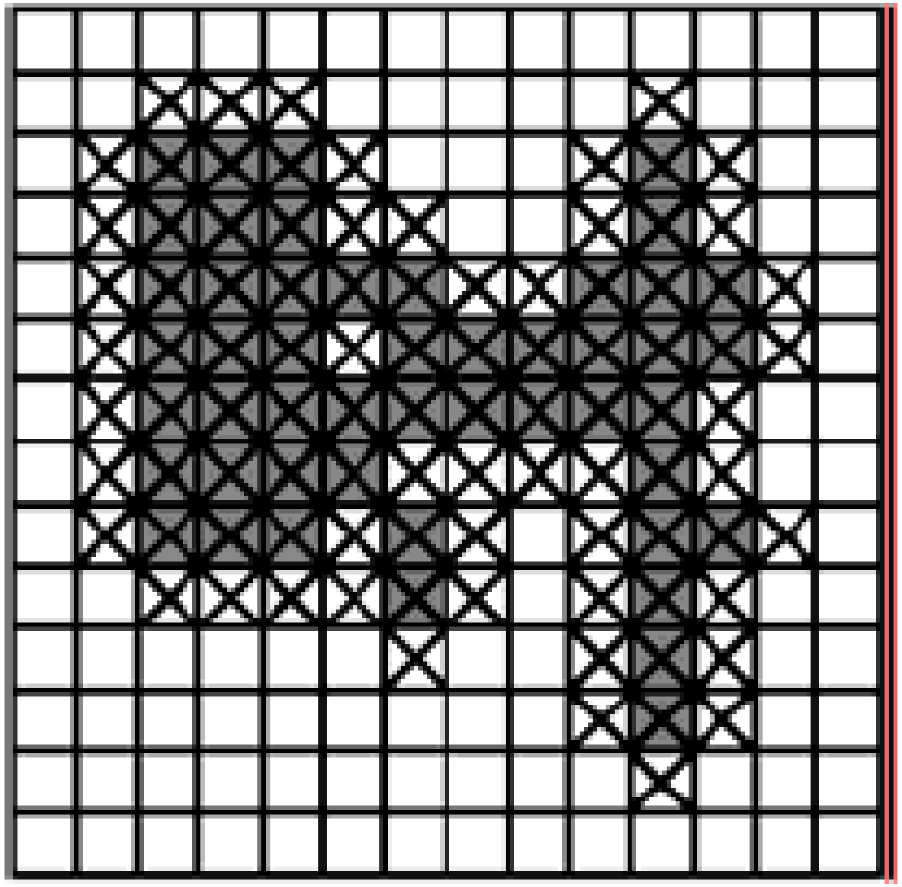
Dilation



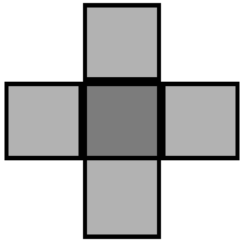
SE=



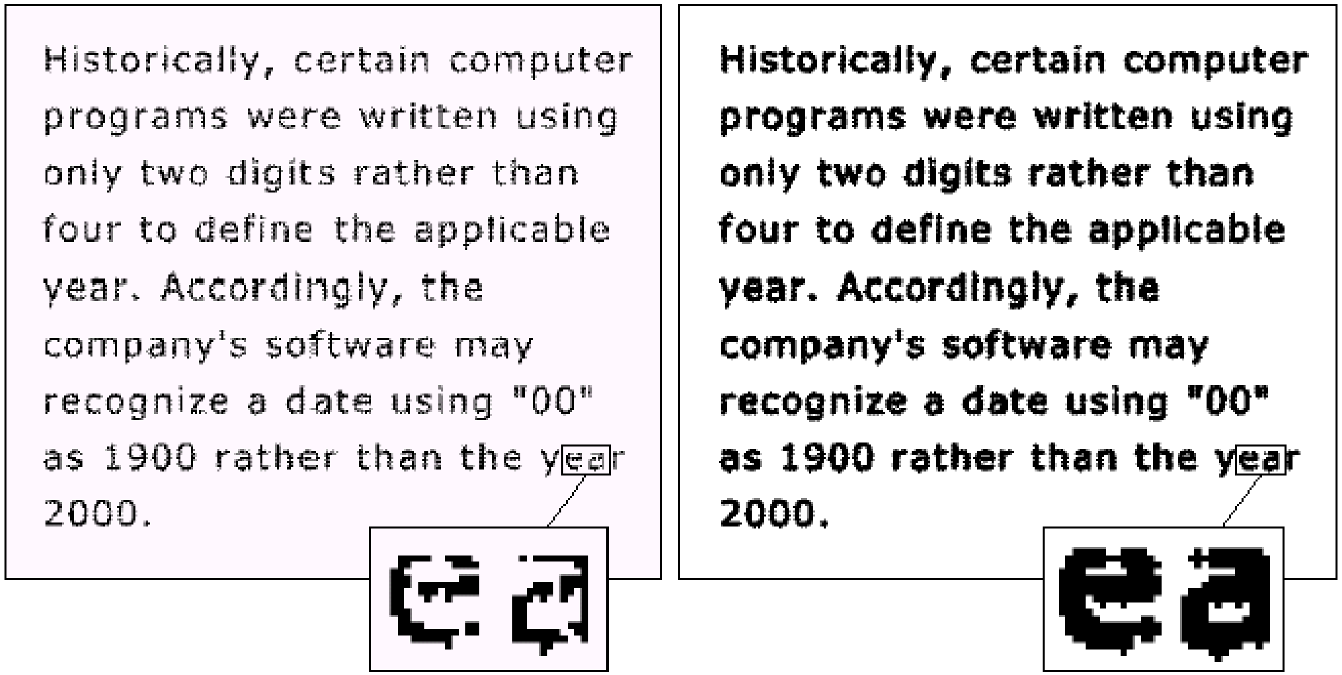
Dilation



SE=



Dilation : Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Useful applications

Erosion

- removal of structures of certain shape and size, given by SE

Dilation

- filling of holes of certain shape and size, given by SE

Combining erosion and dilation

WANTED:

- remove structures / fill holes
- without affecting remaining parts

SOLUTION:

combine erosion and dilation

(using same SE)

Erosion : eliminating irrelevant detail

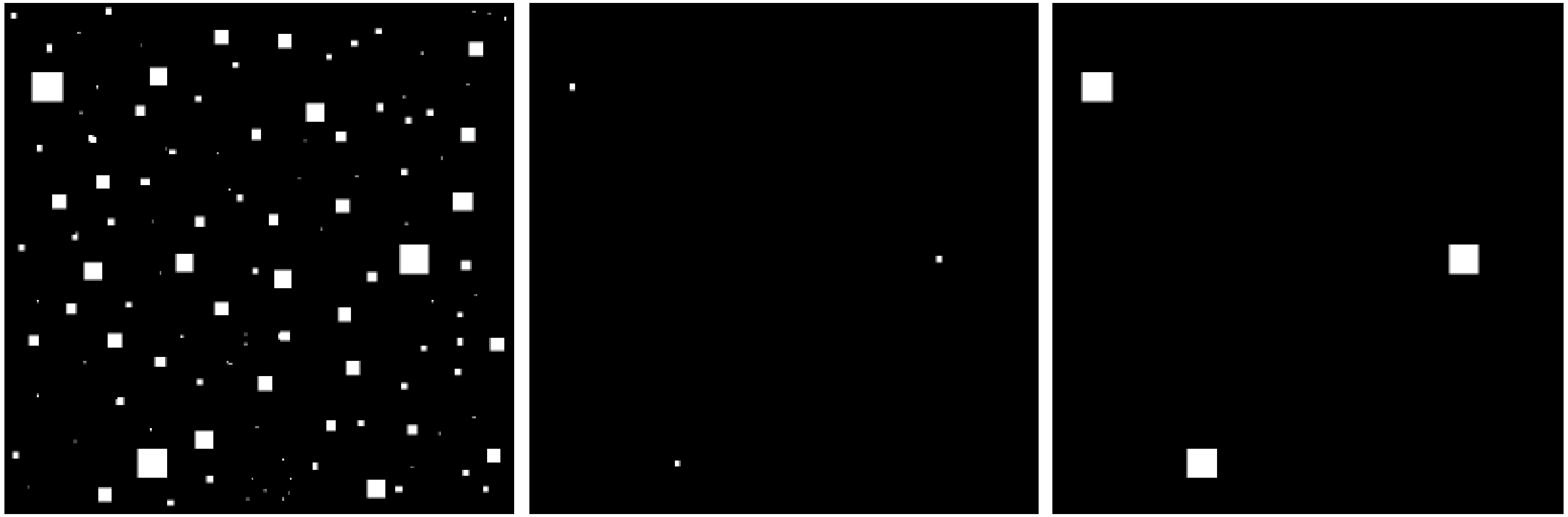


FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of white

Opening

erosion followed by dilation, denoted ∘

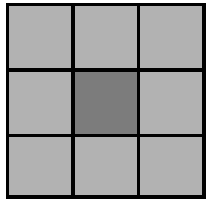
$$A \circ B = (A \ominus B) \oplus B$$

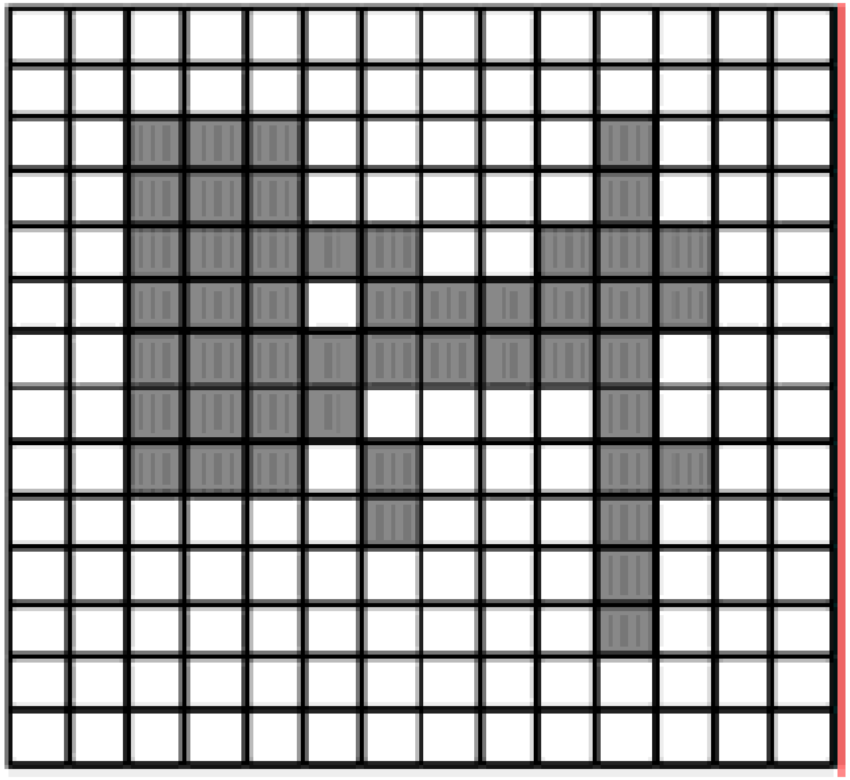
eliminates protrusions

breaks necks

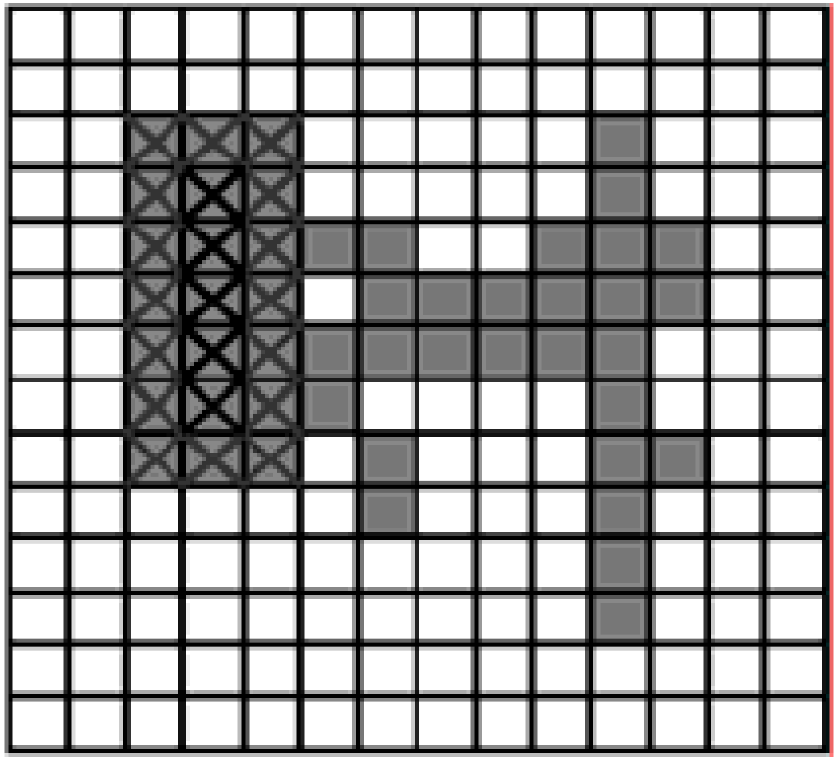
smoothes contour

Opening

B= 



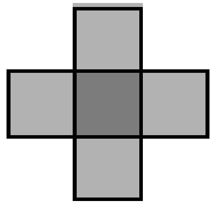
A

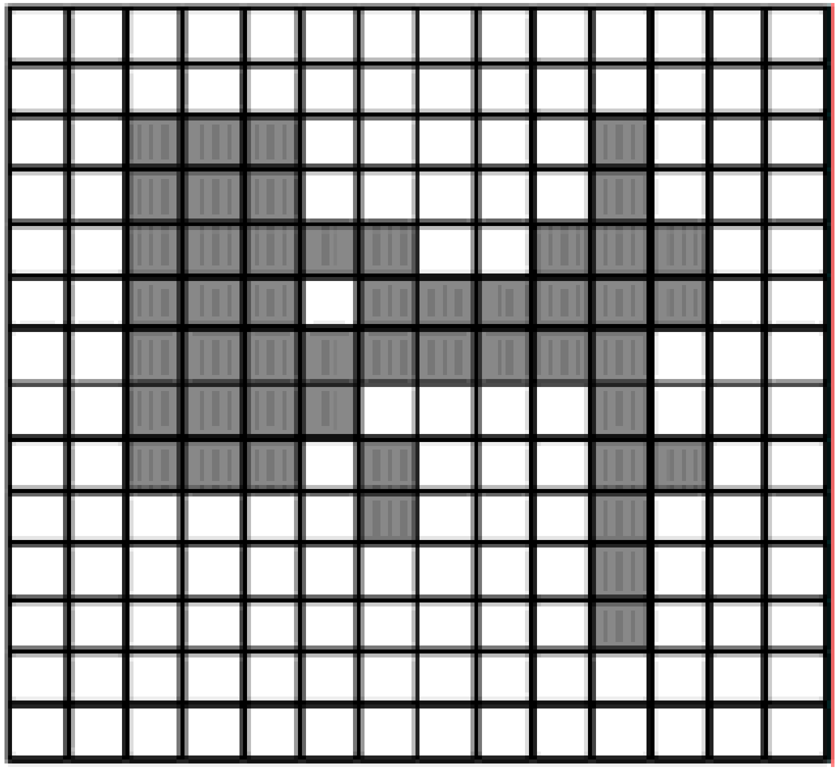


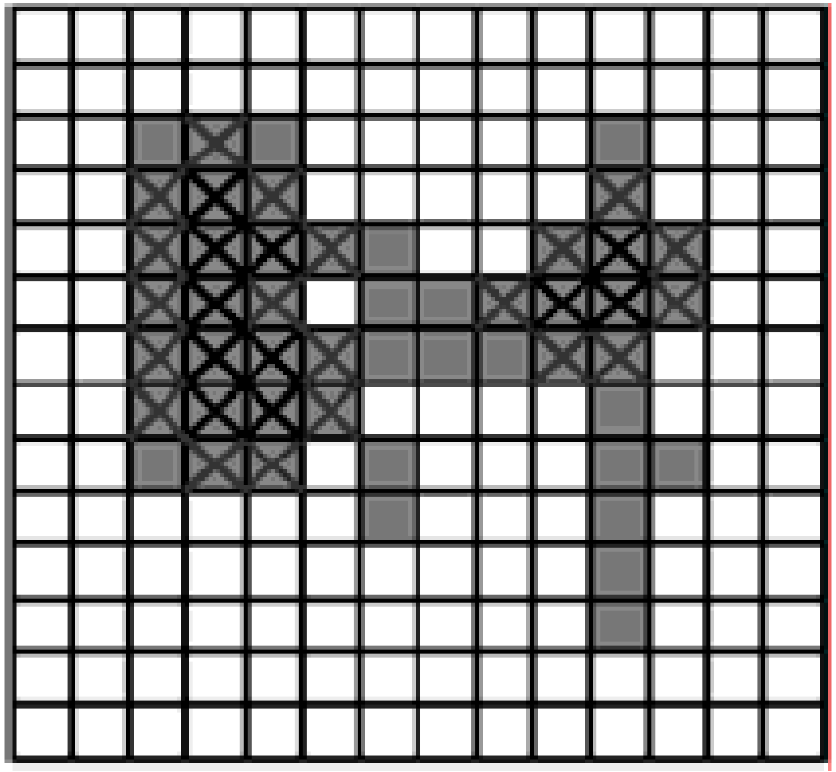
A⊖B A∘B

Opening

B=



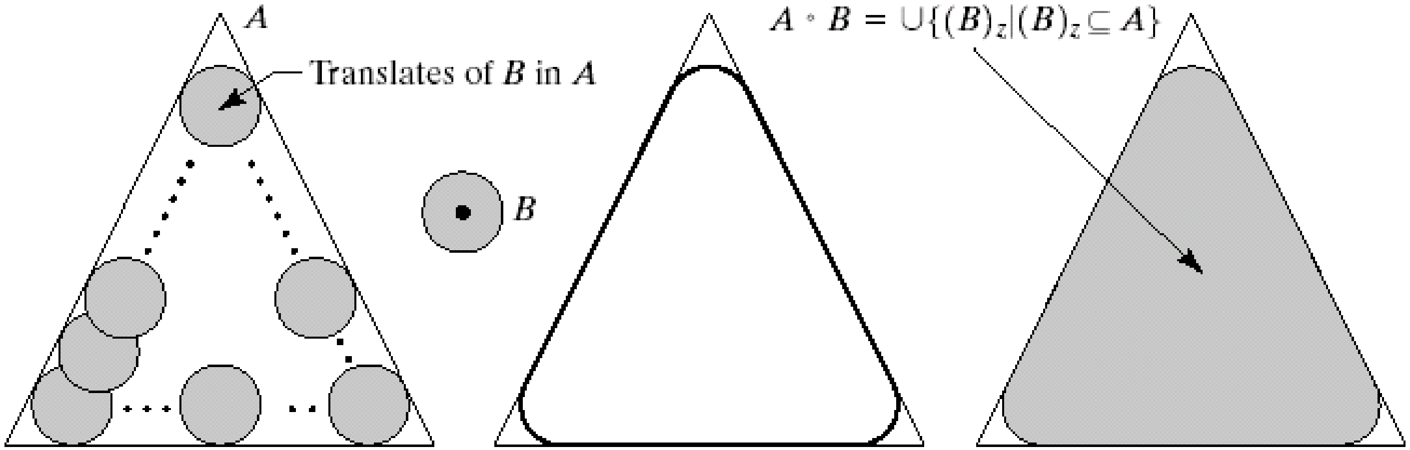




A

A⊖B A∘B

Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

Closing

dilation followed by erosion, denoted •

$$A \bullet B = (A \oplus B) \ominus B$$

smooth contour

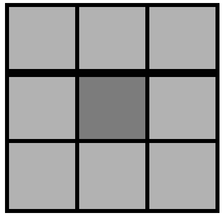
fuse narrow breaks and long thin gulfs

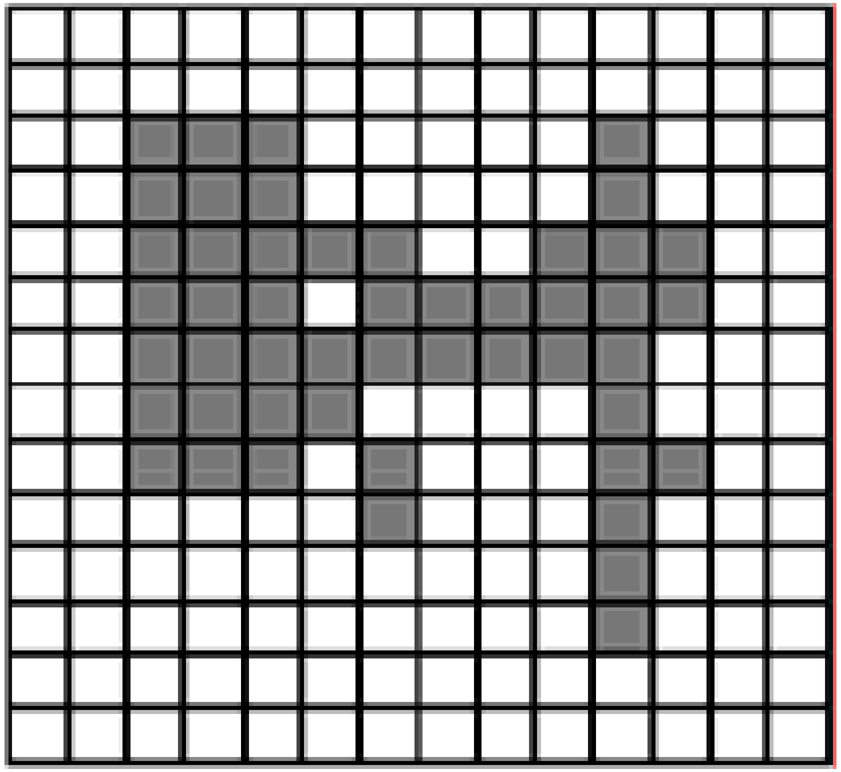
eliminate small holes

fill gaps in the contour

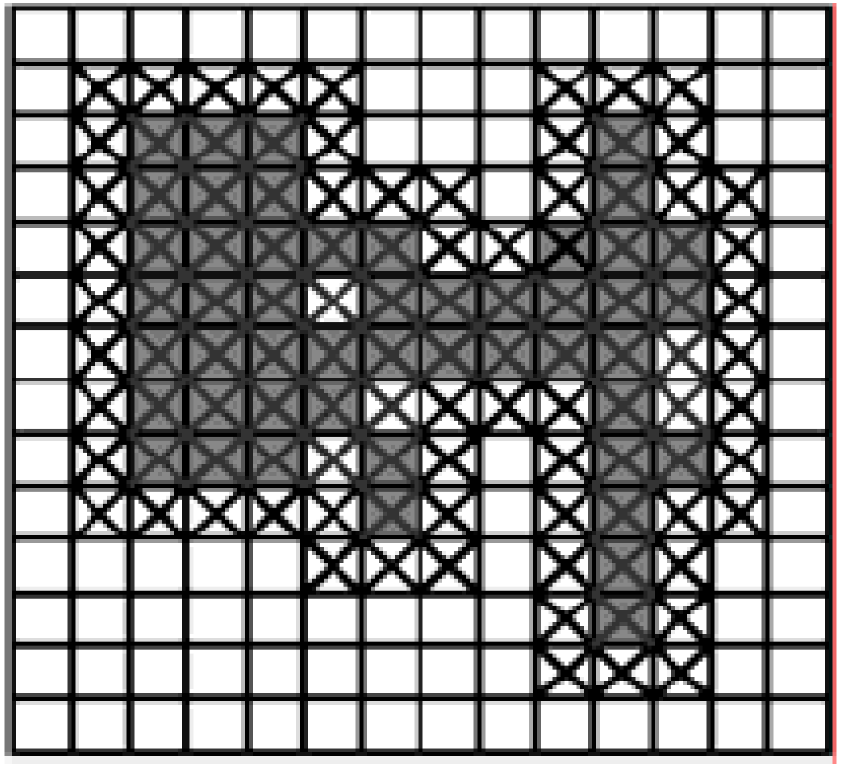
Closing

B=



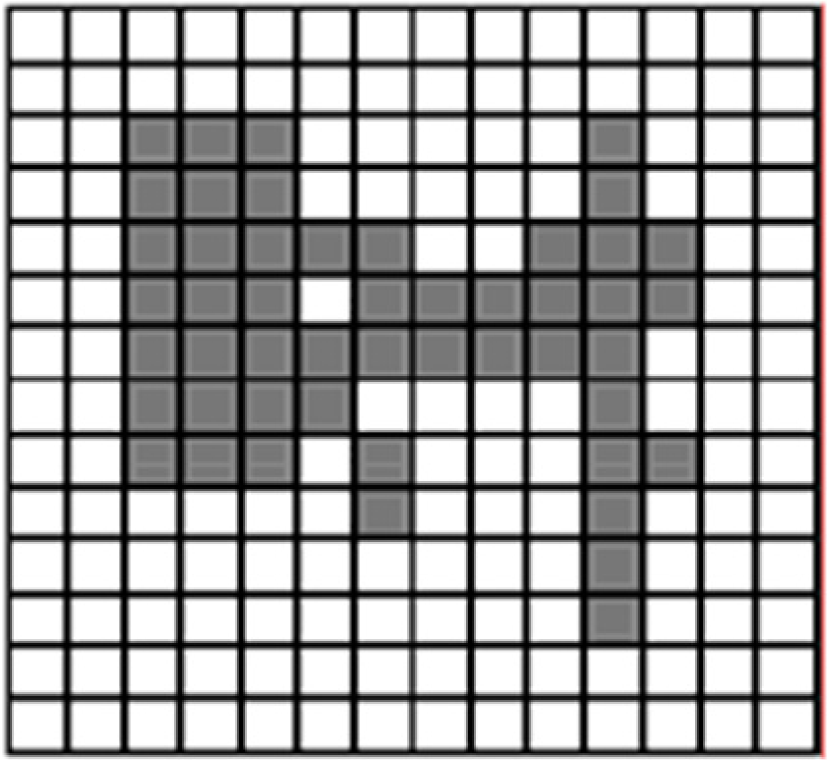


A



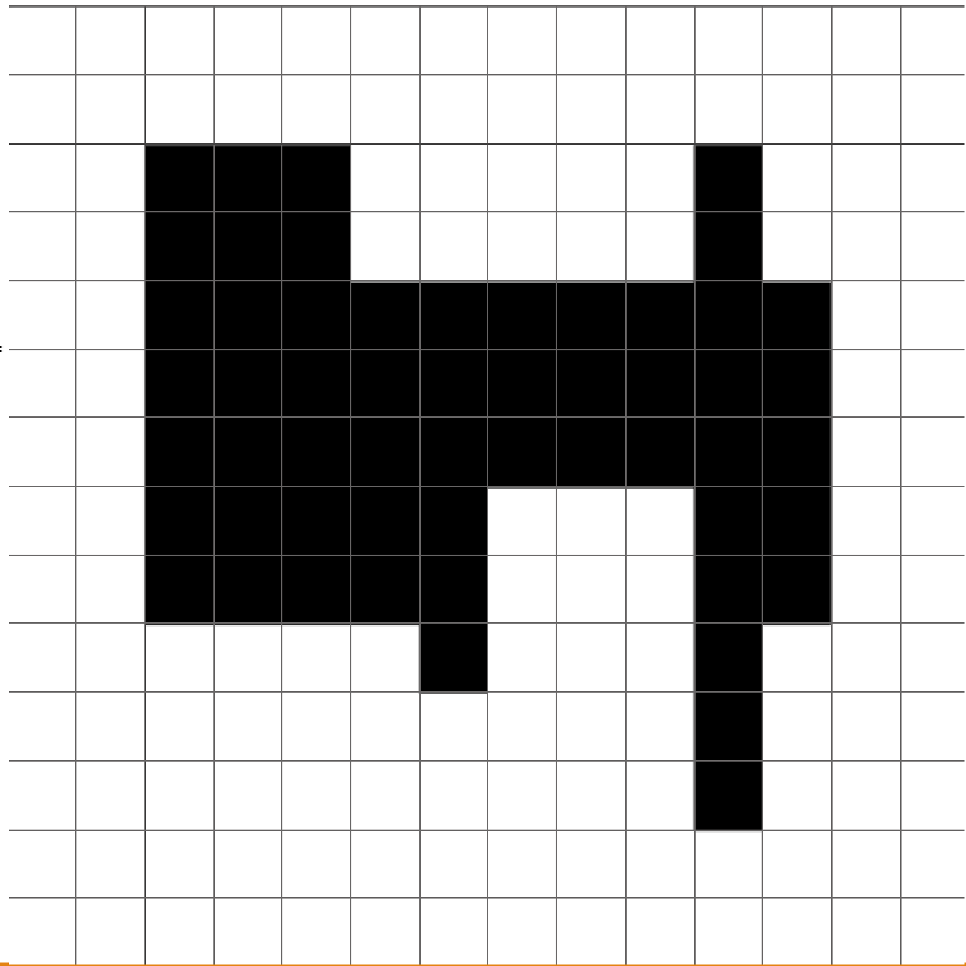
A⊕B

Closing final result



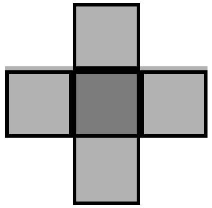
A

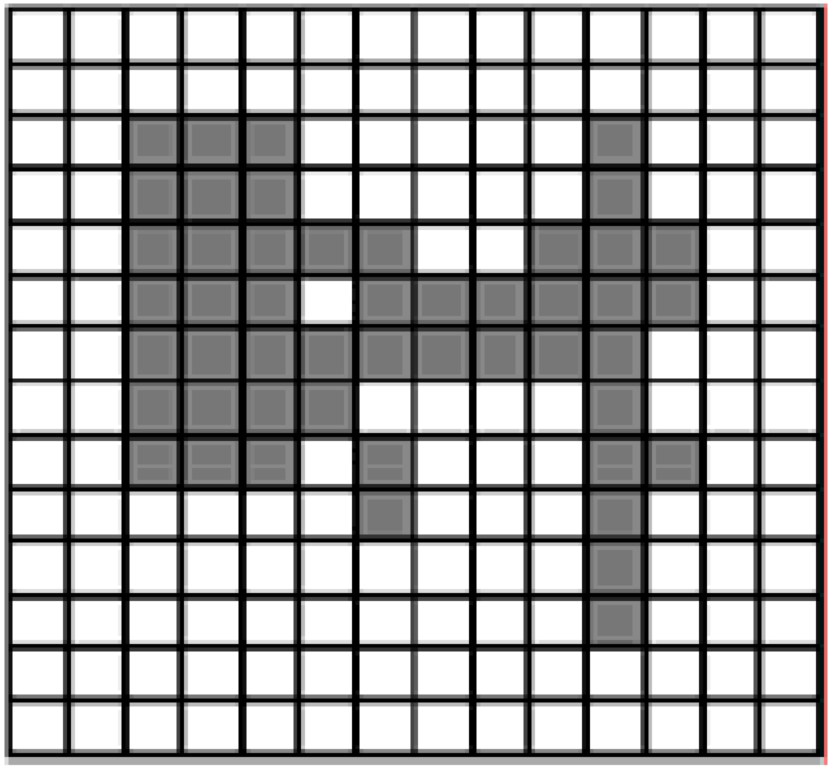
B=



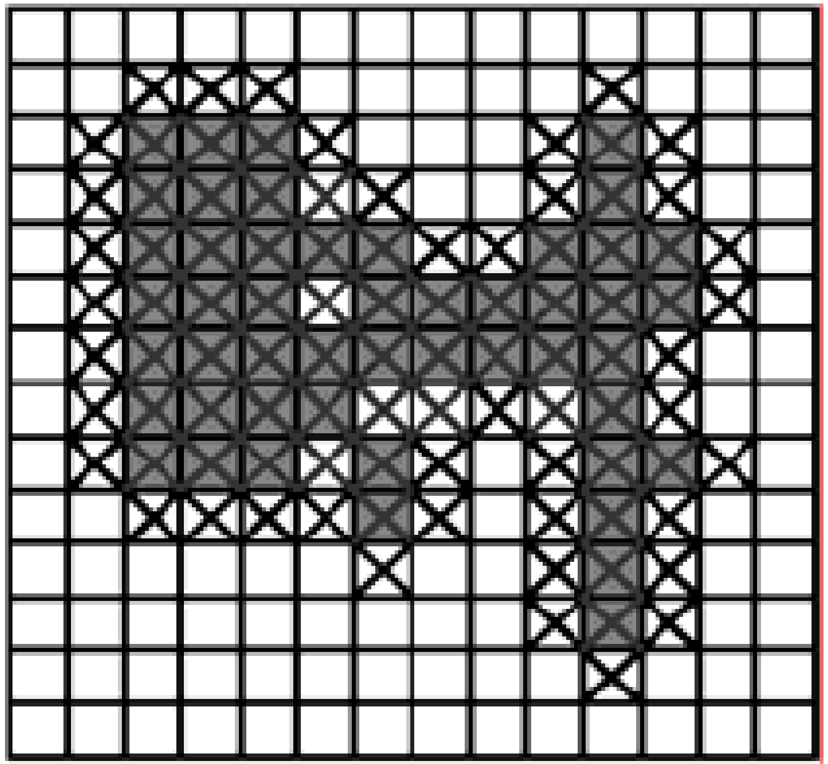
Closing

B=





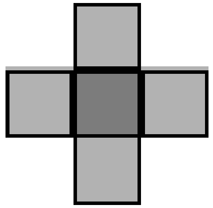
A

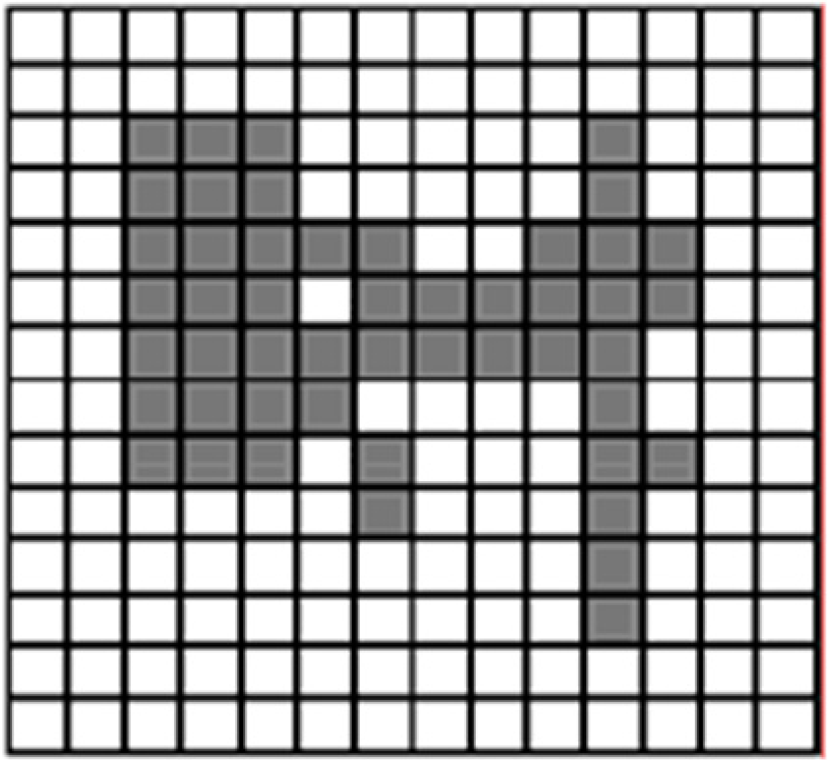


A⊕B

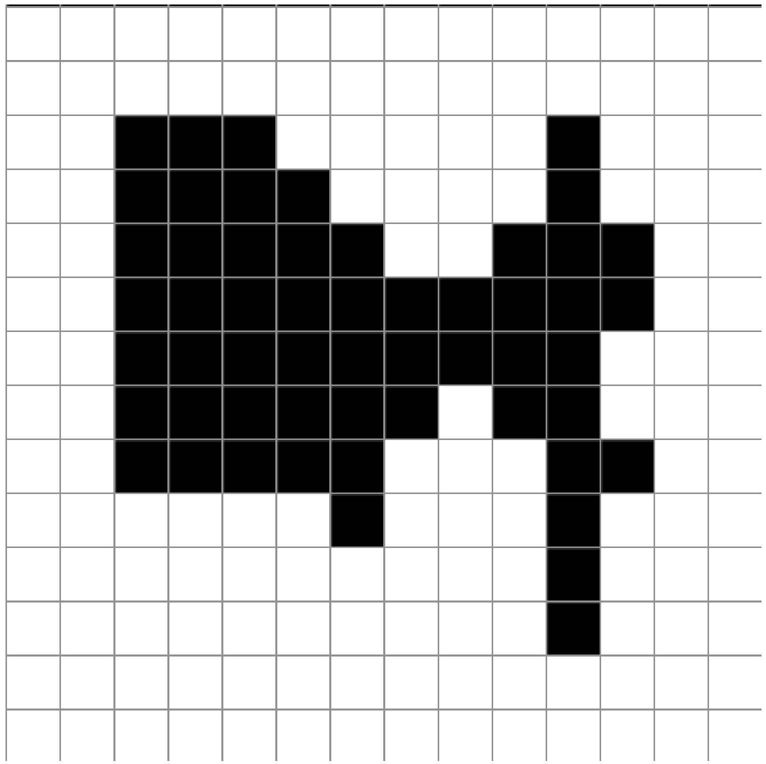
Closing final result

B=





A

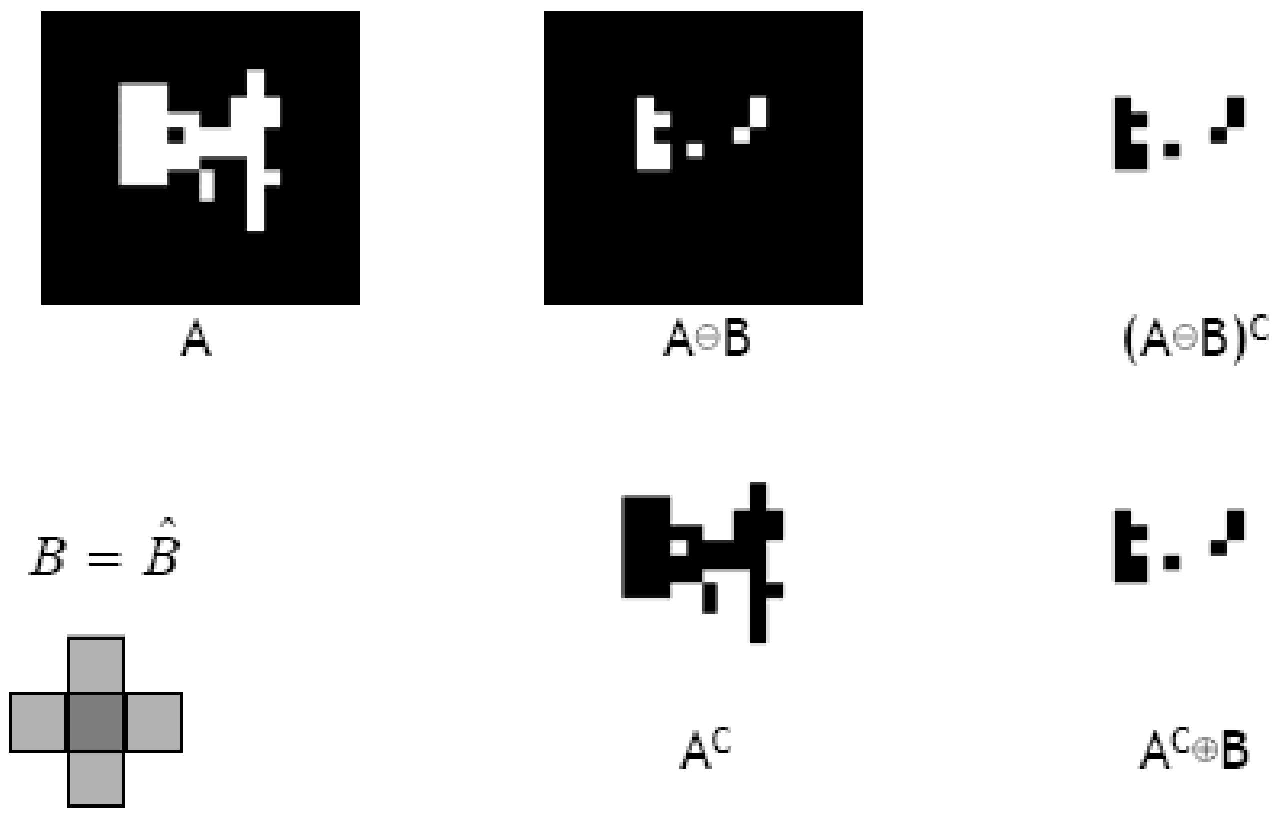


Duality

Opening and closing are dual with respect to complementation and reflection

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Duality



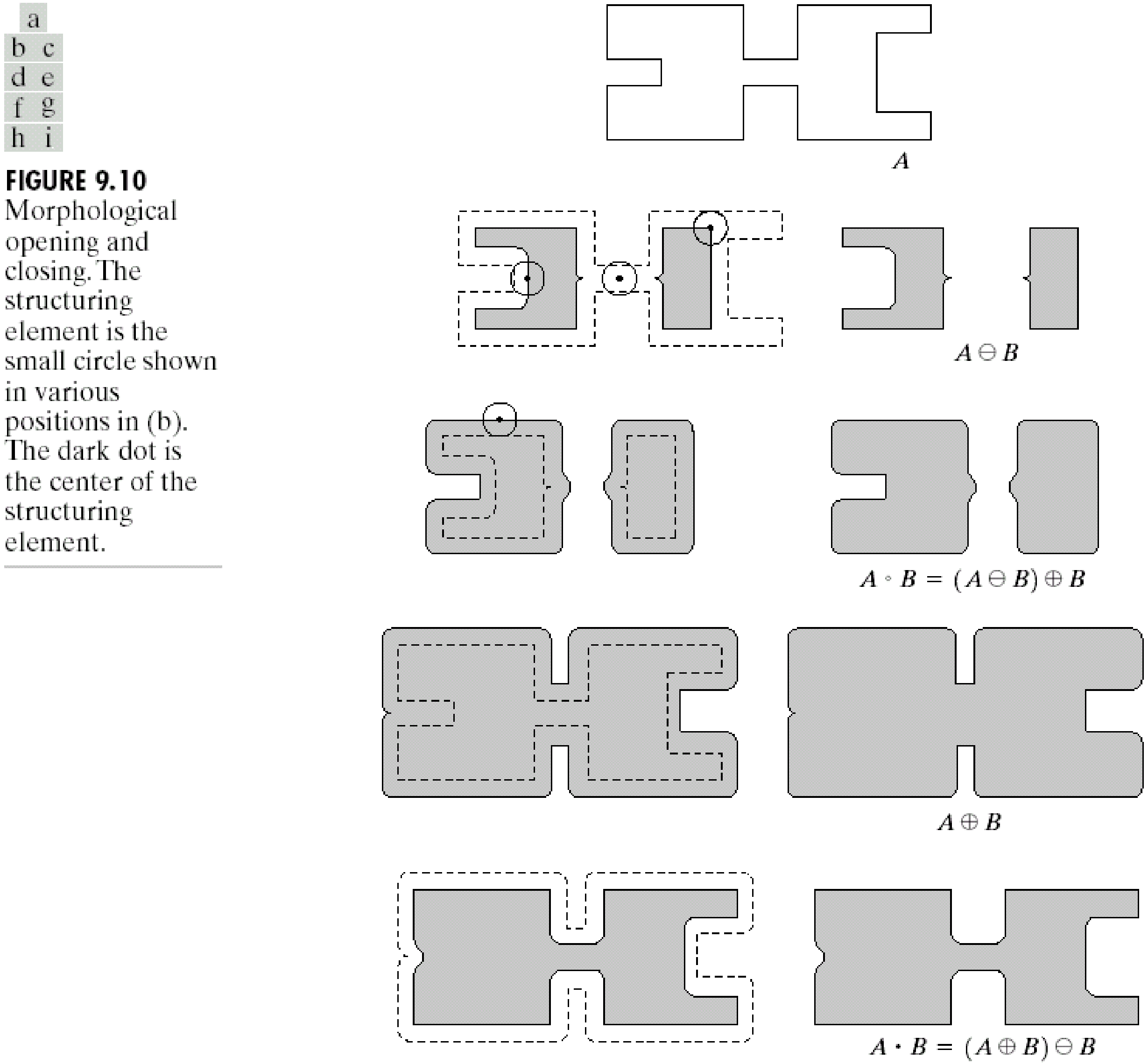
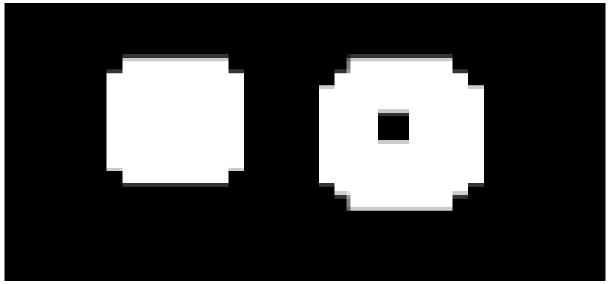


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Useful: open & close

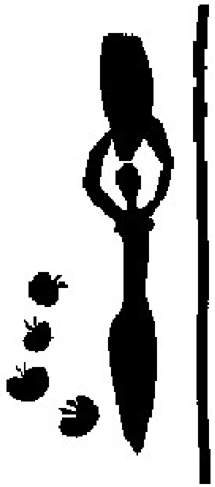
A



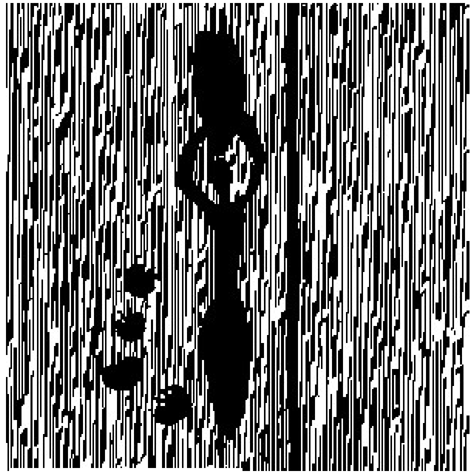
opening of A
→removal of small protrusions, thin connections, ...

closing of A
→ removal of holes

Filtering example



ORIGINAL

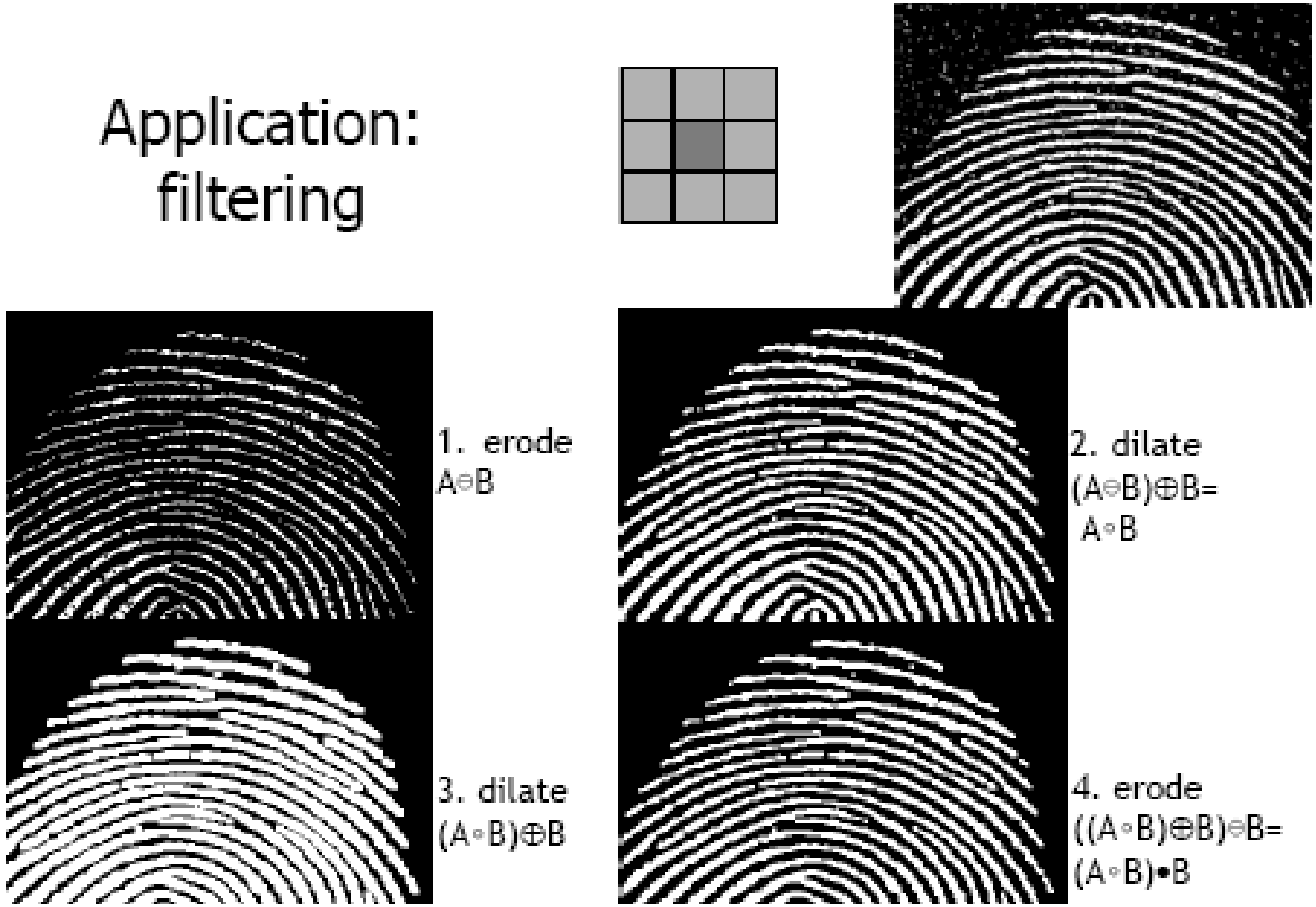


DEGRADED

FILTERED

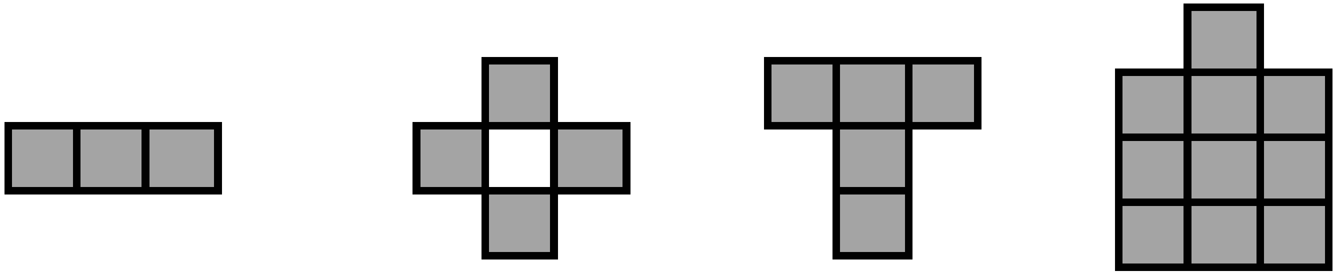
Henri Matisse, *Woman with Amphora and Pomegranates*, 1952

Application: filtering



Hit-or-Miss Transformation ⊛ (HMT)

find location of one shape among a set of shapes
”template matching

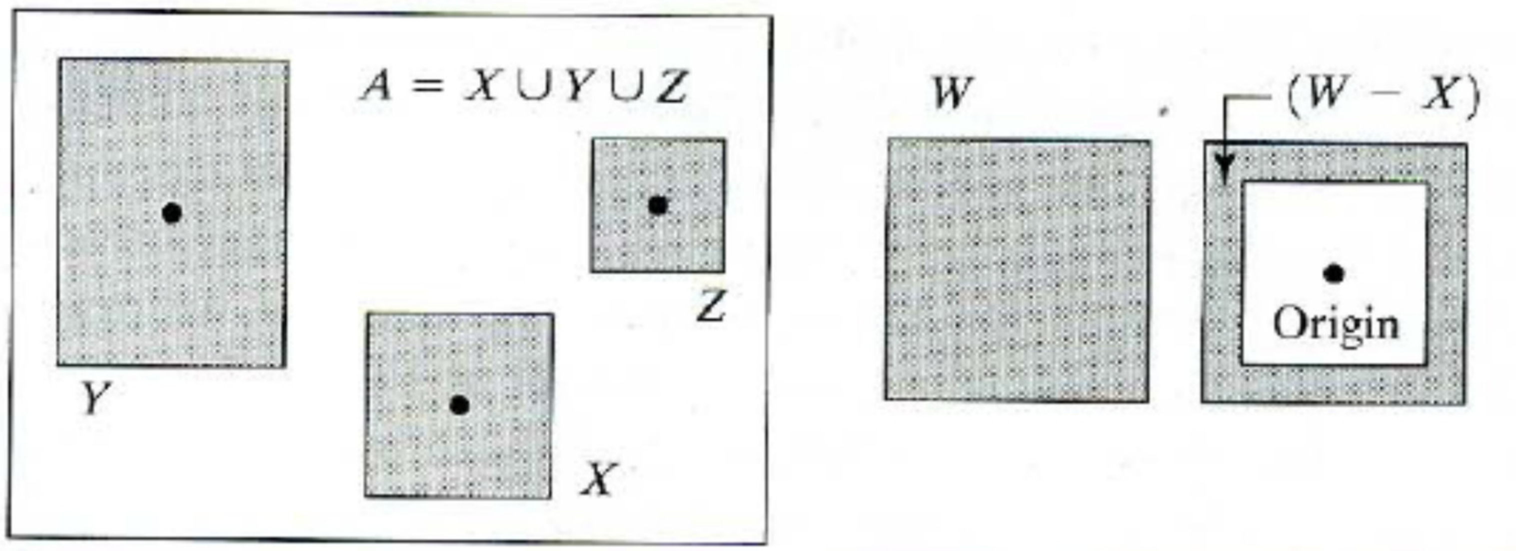


composite SE: object part (B1) and background part (B2)

does B1 ***fits the object while, simultaneously,*** B2 misses the object, i.e., ***fits the background?***

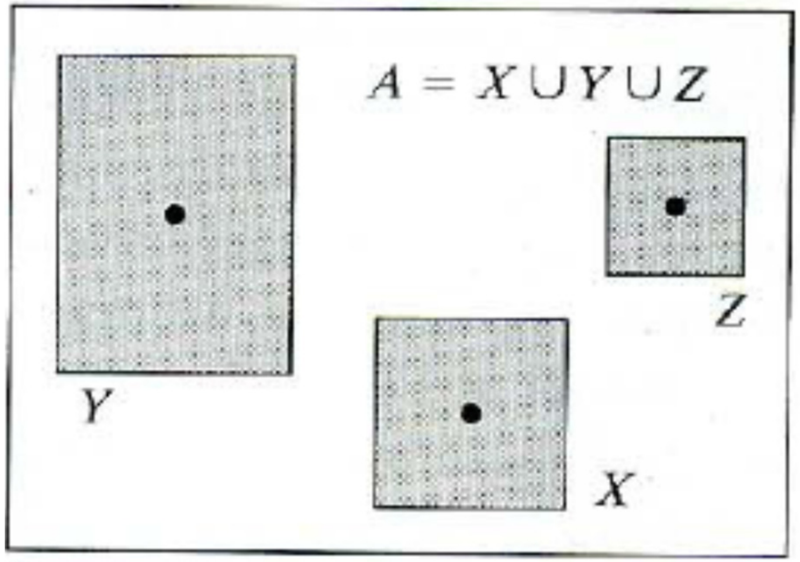
Hit-or-Miss operator

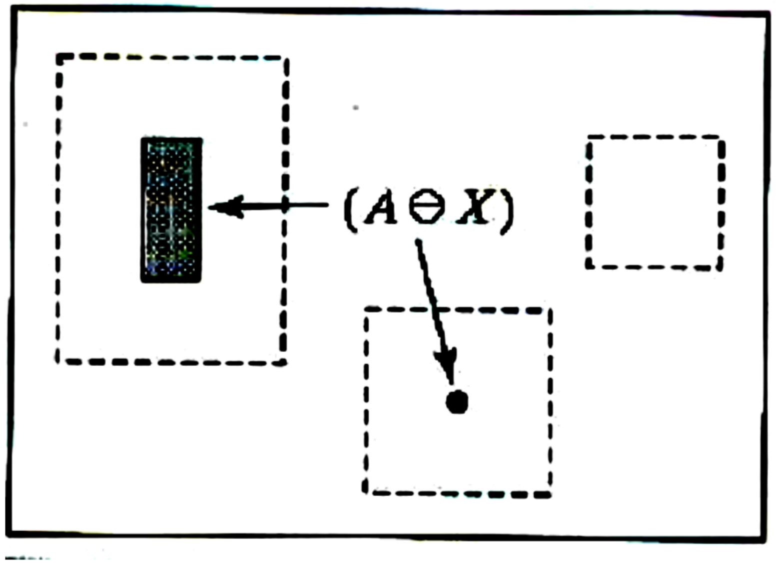
Find an exact shape inside a binary image



Local-background (*W*-*X*)

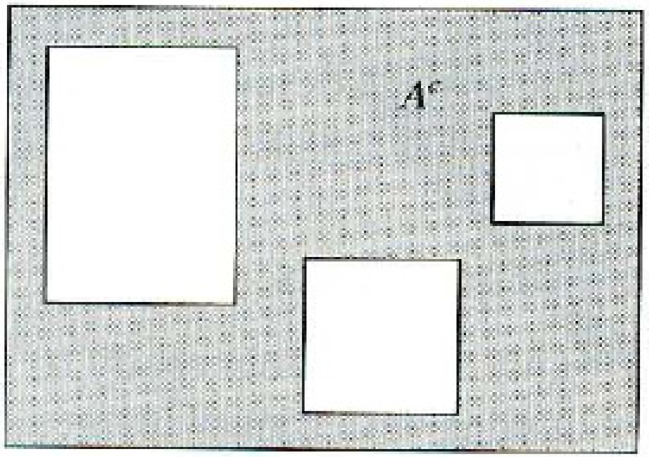
Hit-or-Miss (cont.)

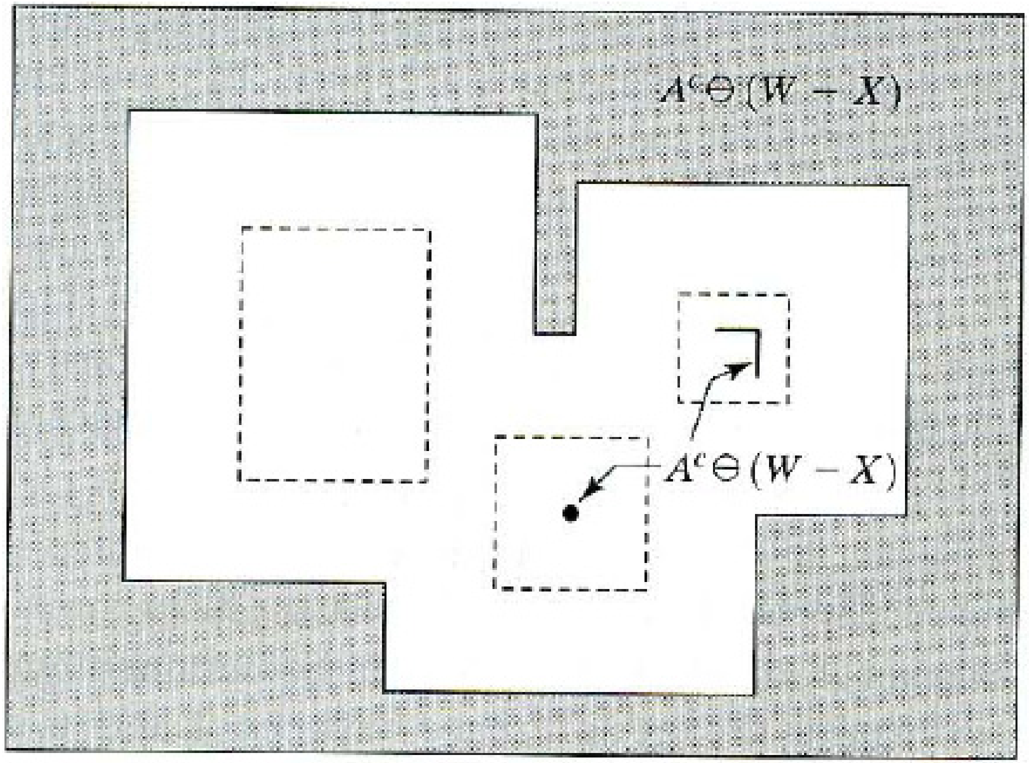




Erosion with the searched shape X

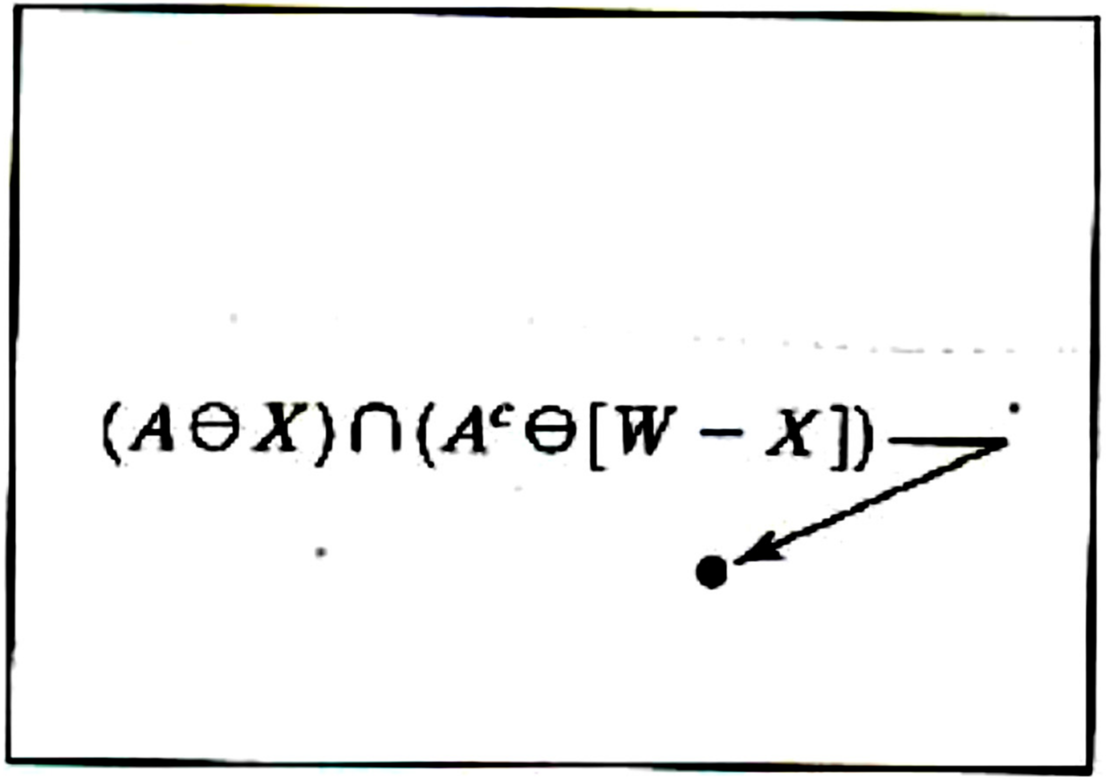
Hit-or-Miss (cont.)





Erosion of A^C with the Local Background

Hit-or-Miss (cont.)



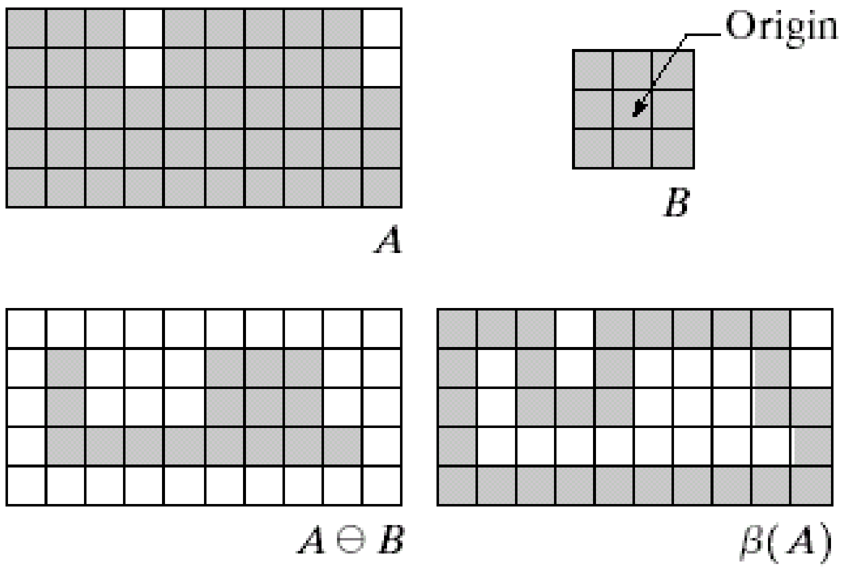
Intersection between the two erosions

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

Boundary Extraction

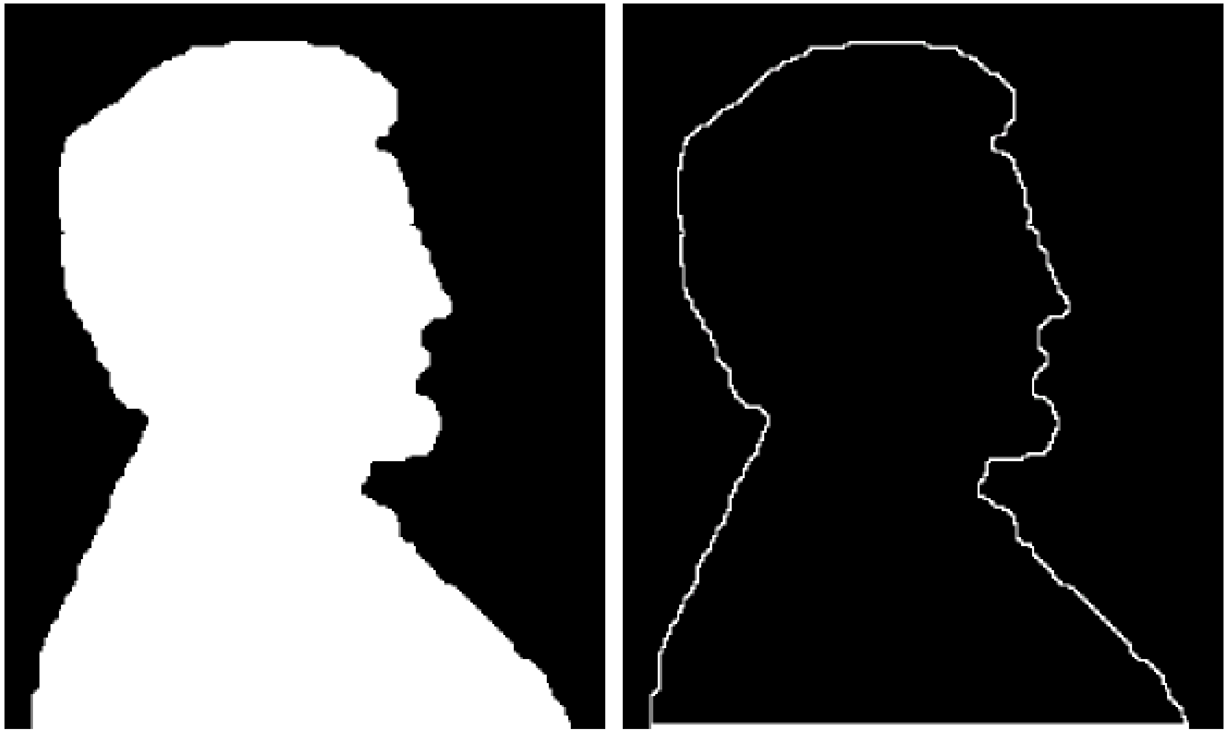
a	b
c	d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



$$\beta(A) = A - (A \ominus B)$$

Example



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$

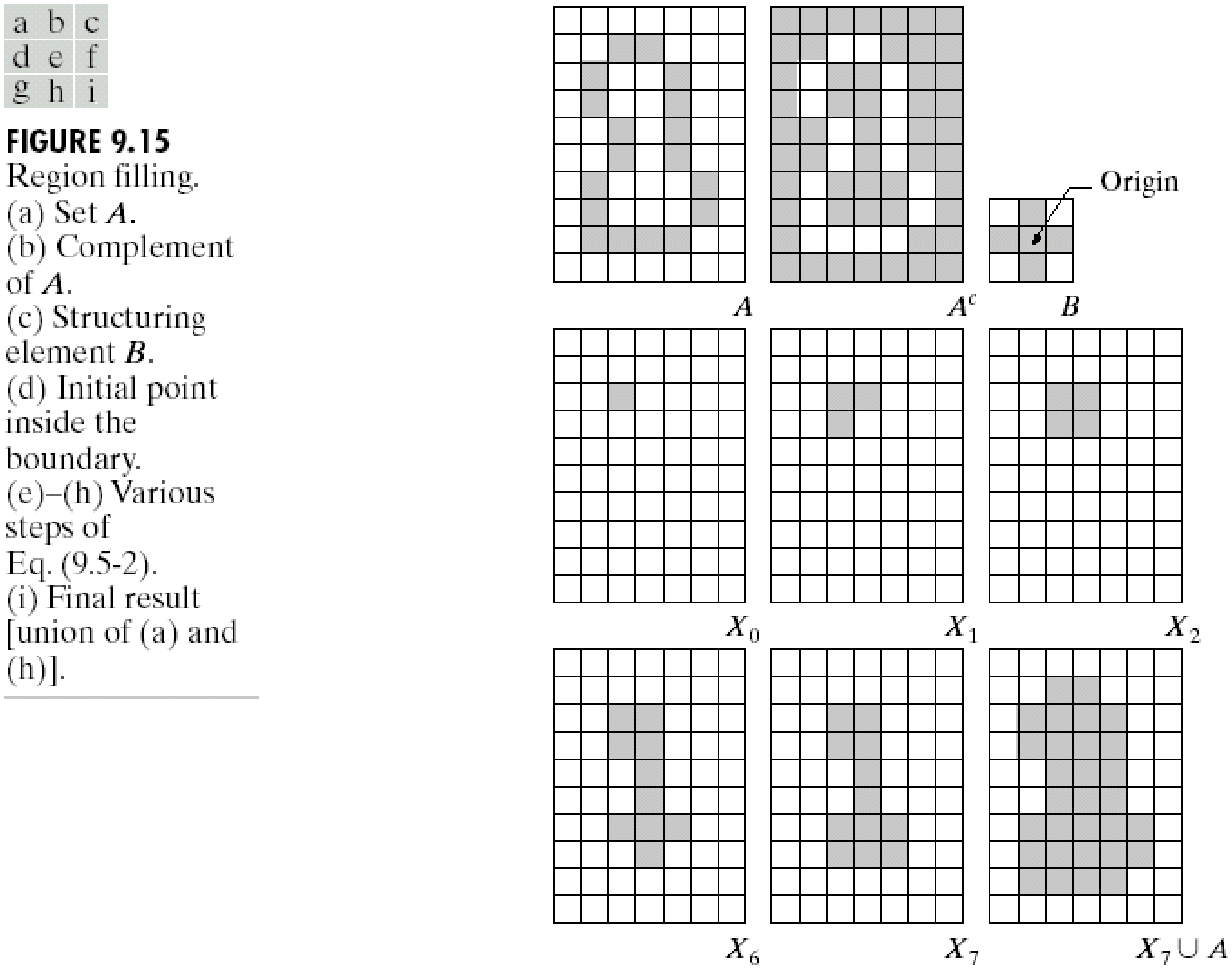
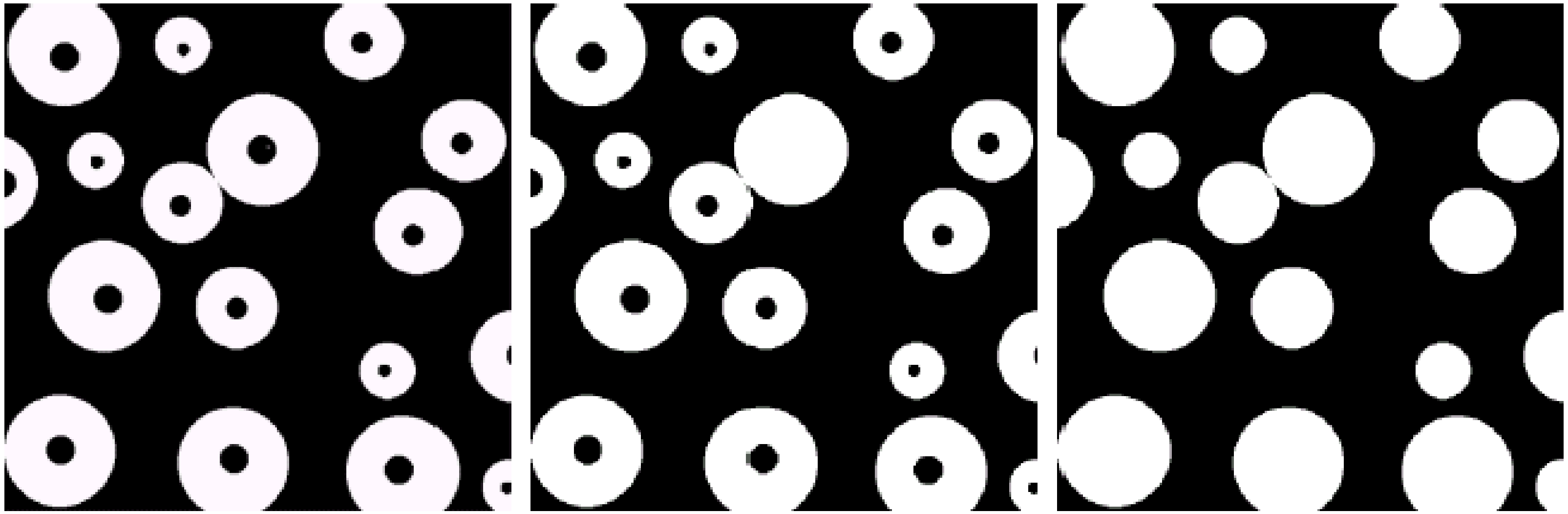


FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extraction of connected components

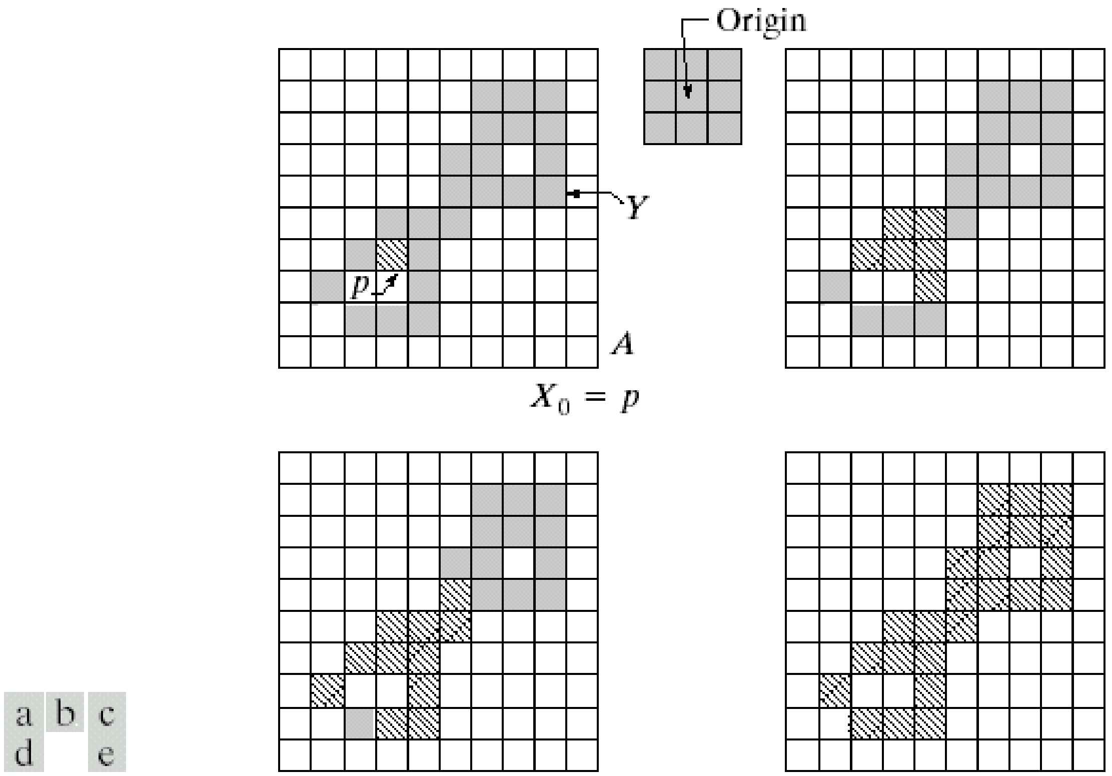


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

Example

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

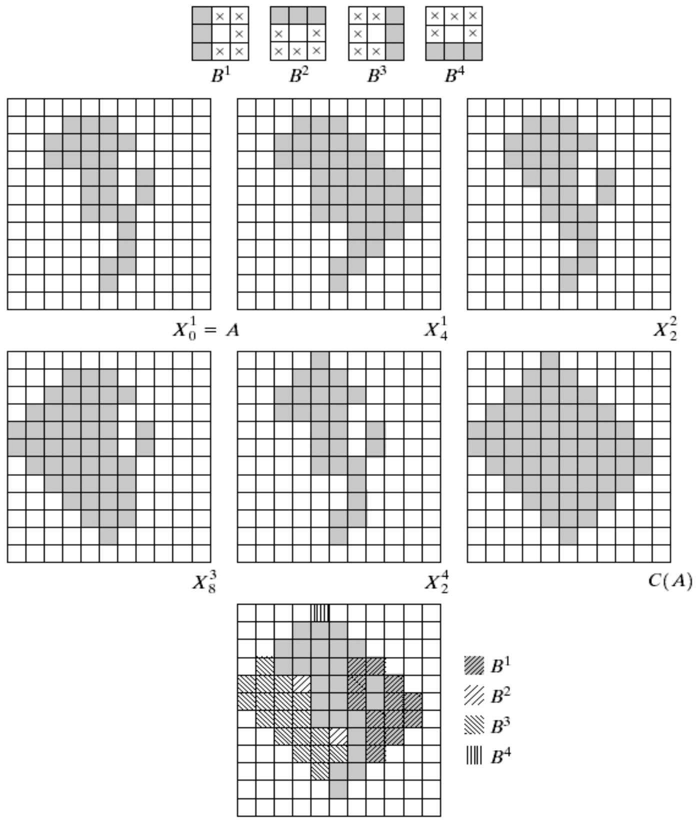
Convex hull

A set *A* is said to be convex if the straight line segment joining any two points in A lies entirely within A.

$$X_k^i = (X_k^i \circledast B^i) \cup A \quad i = 1,2,3,4 \ \text{ and } k = 1,2,3,\ldots$$

$$C(A) = \bigcup_{i=1}^{4} D^i$$

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



Convex hull

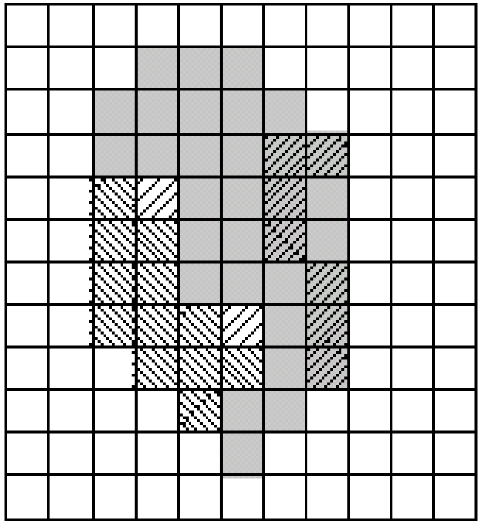


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Thinning

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

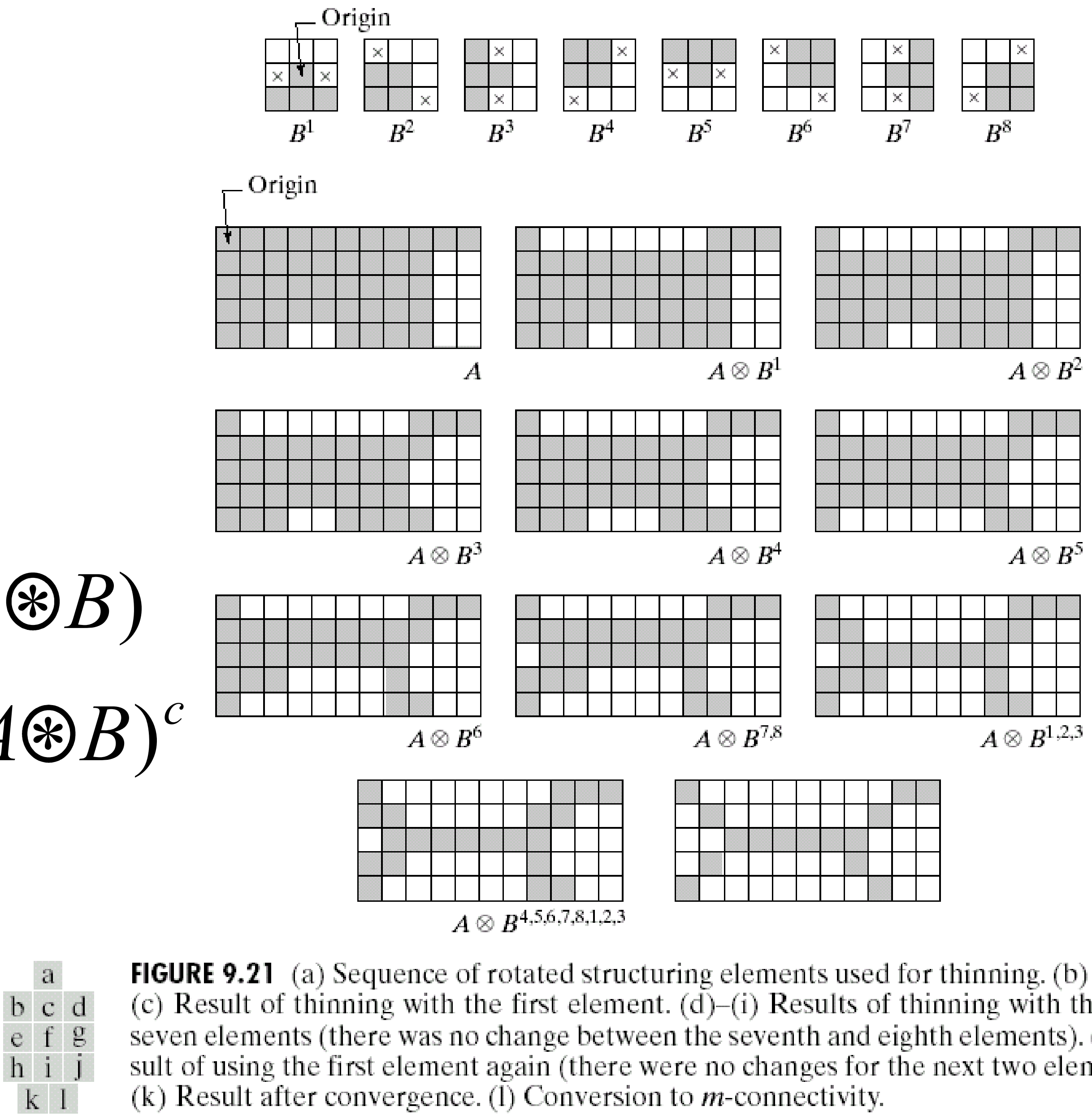
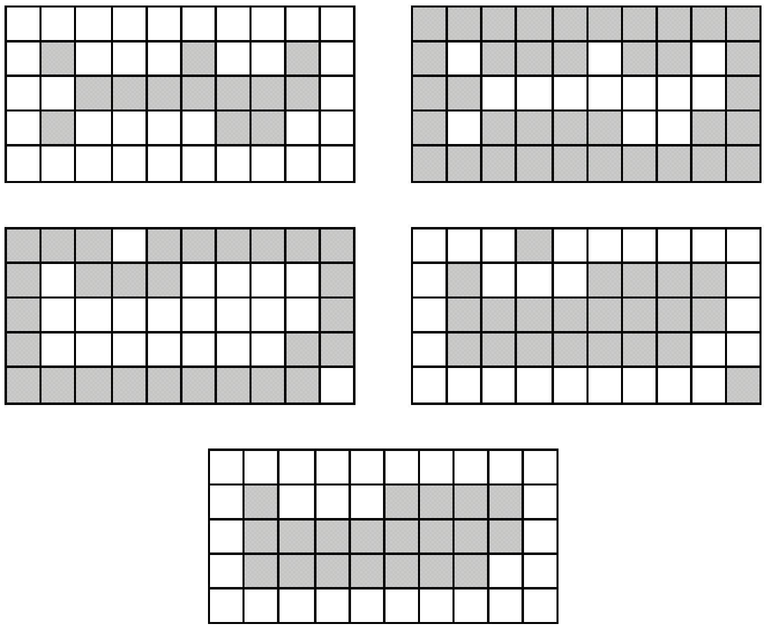


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

$$A \odot B = \left(A^C \cup \left(A^C \circledast B \right) \right)^C$$



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

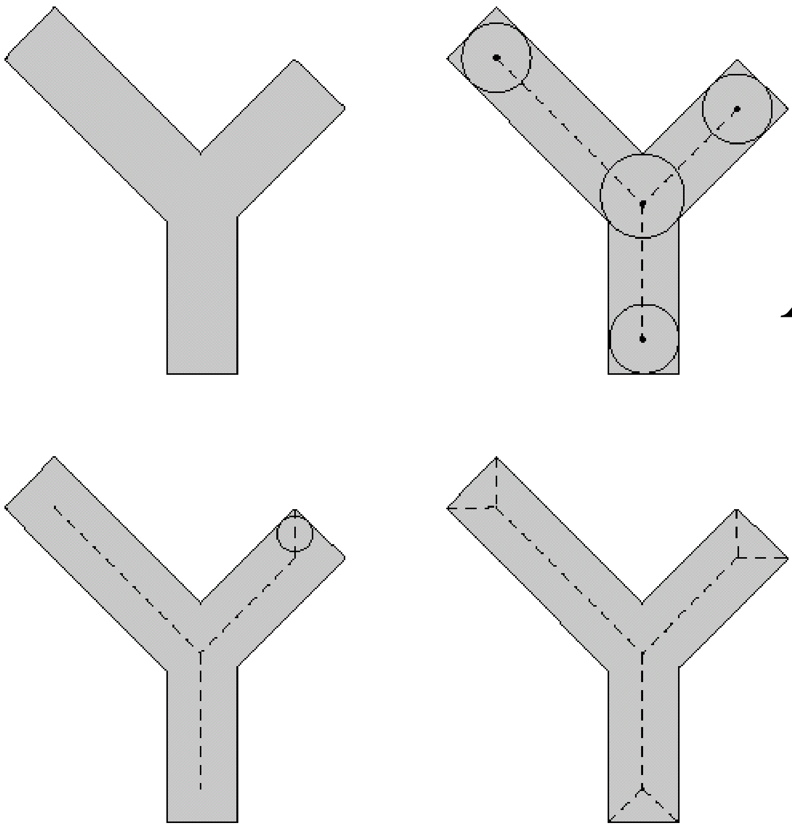
$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.



$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

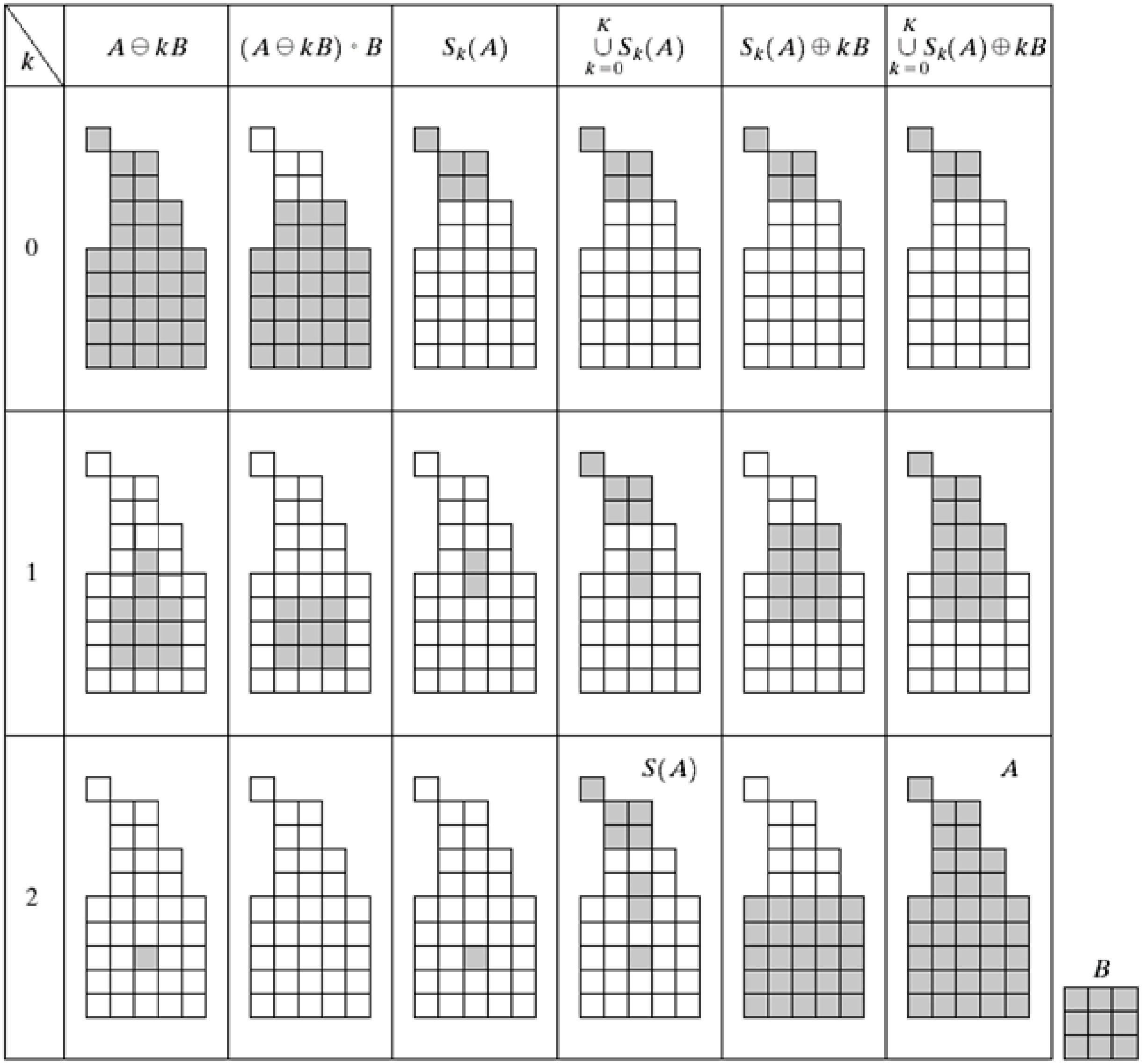


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

Gray scale Dilation

TRANSLATION INVARIANCE

$$\Psi_\delta(F(x-h)) = [\Psi_\delta(F)](x-h)$$

$$\Psi_\delta(F(x)+v) = [\Psi_\delta(F)](x)+v$$

"LINEARITY"

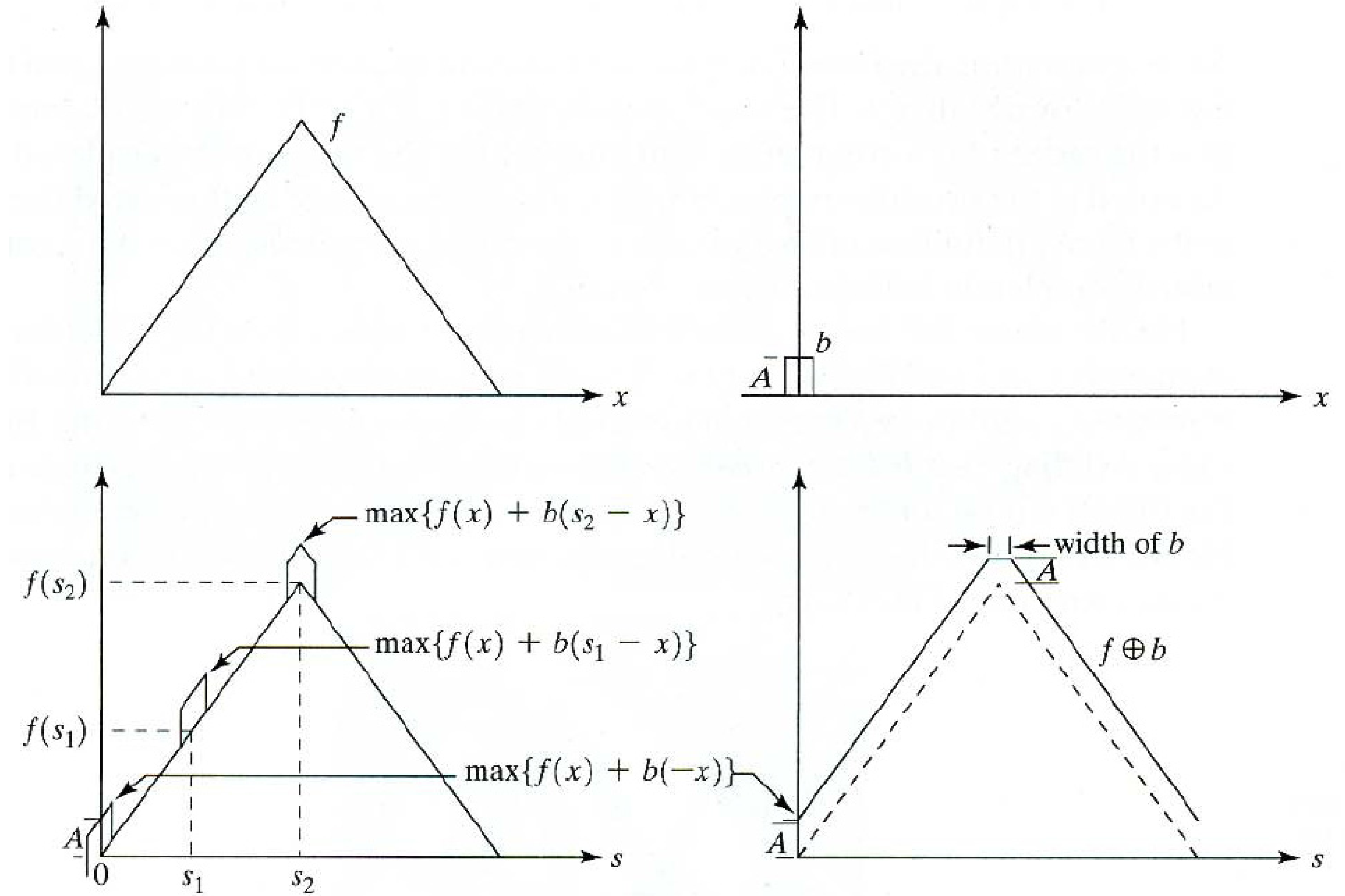
$$\Psi_\delta(F_1 \vee F_2) = \Psi_\delta(F_1) \vee \Psi_\delta(F_2)$$

MAXIMUM

$$[\Psi_\delta(F)](x) = F \oplus B(x) = \bigvee_h [F(h) + B(x-h)]$$

Gray scale Dilation

$$(f \oplus b)(s,t) = \max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f;\ (x,y) \in D_b\}$$



Gray scale Erosion

TRANSLATION INVARIANCE

$$\Psi_\varepsilon(F(x-h)) = [\Psi_\varepsilon(F)](x-h)$$

$$\Psi_\varepsilon(F(x)+v) = [\Psi_\varepsilon(F)](x)+v$$

"LINEARITY"

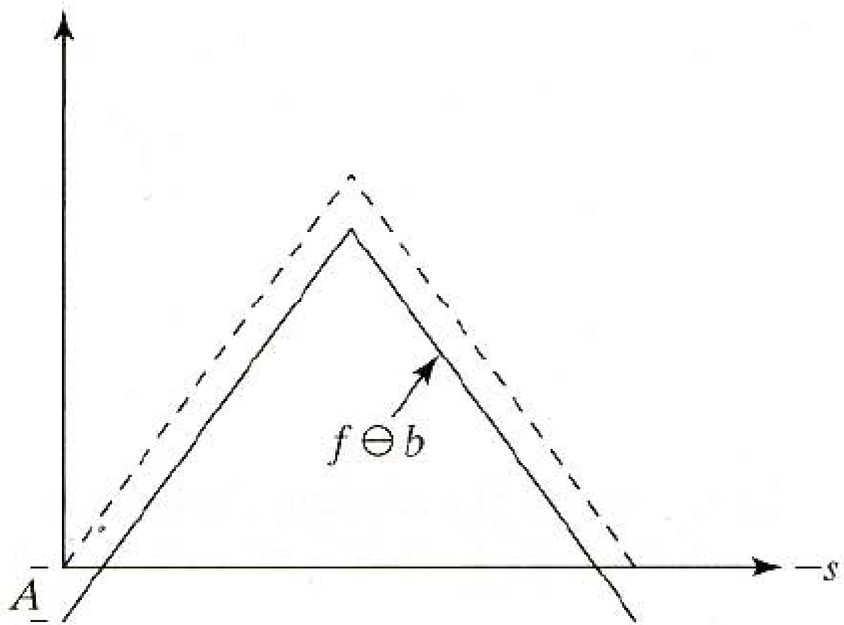
$$\Psi_\varepsilon(F_1 \wedge F_2) = \Psi_\varepsilon(F_1) \wedge \Psi_\varepsilon(F_2)$$

MINIMUM

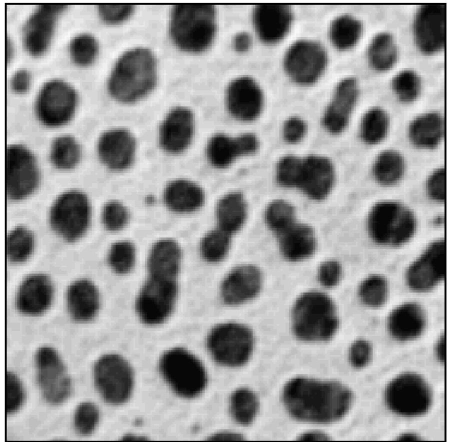
$$[\Psi_\varepsilon(F)](x) = F \ominus B(x) = \bigwedge_h [F(h) - B(h-x)]$$

Gray scale erosion

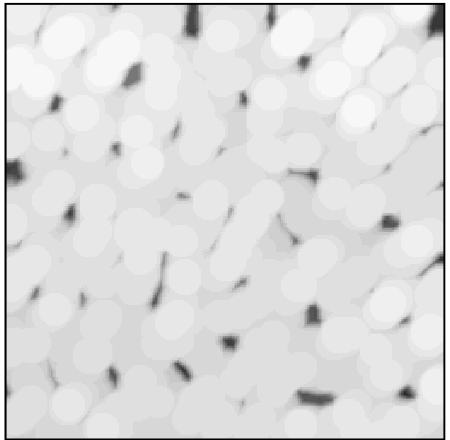
$(f \ominus b)(s, t)$
$\min\{f(s +$ $D_f; (x, y) \in D_b\}$



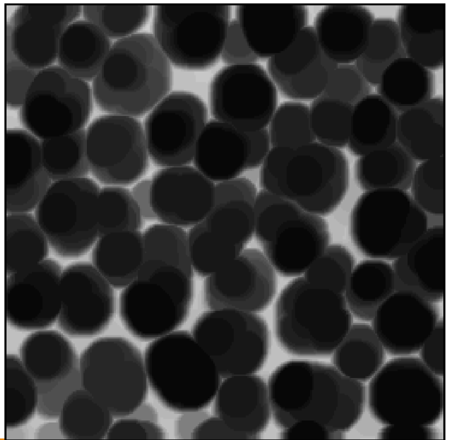
Gray scale morphology



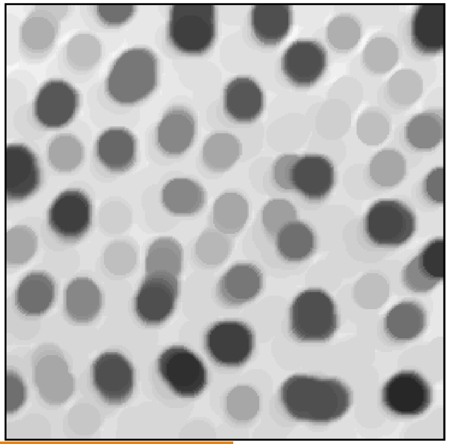
ORIGINAL



EROSION

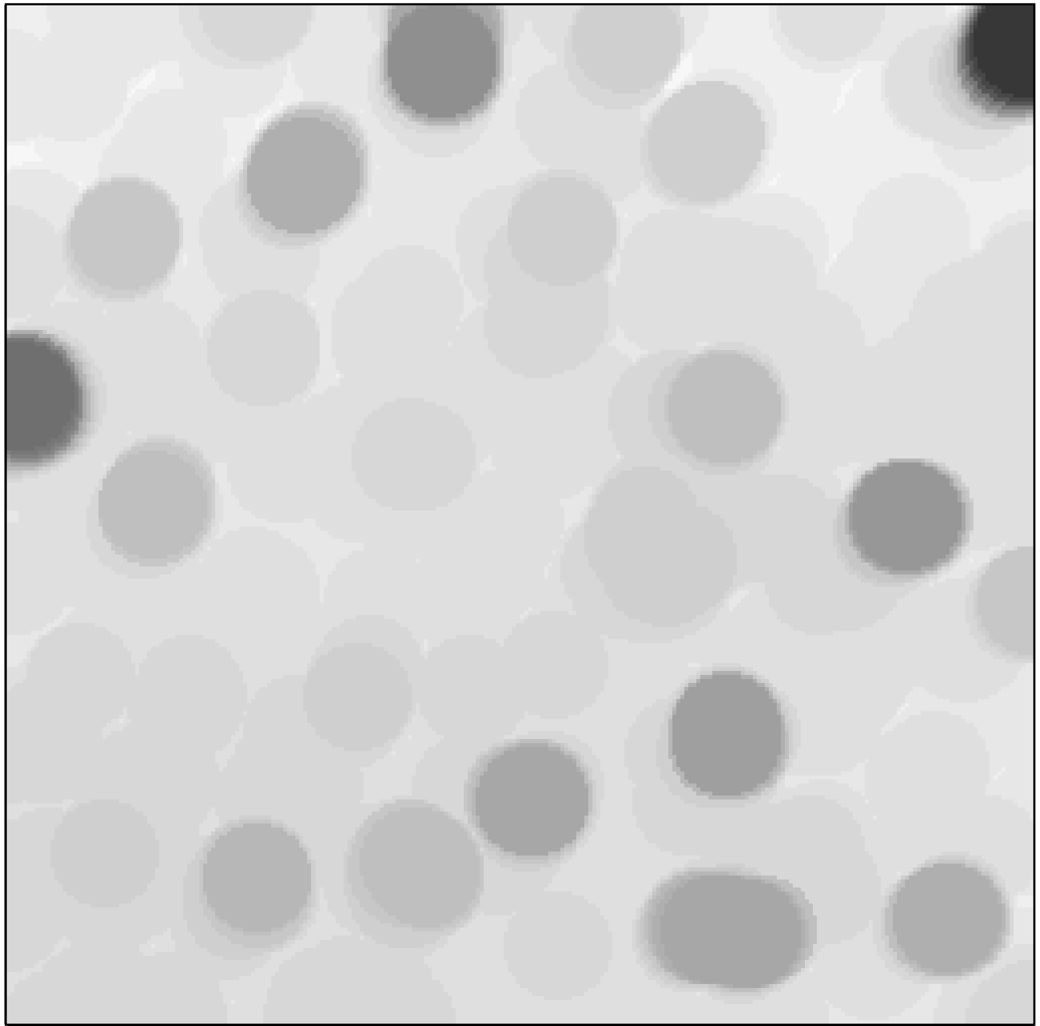


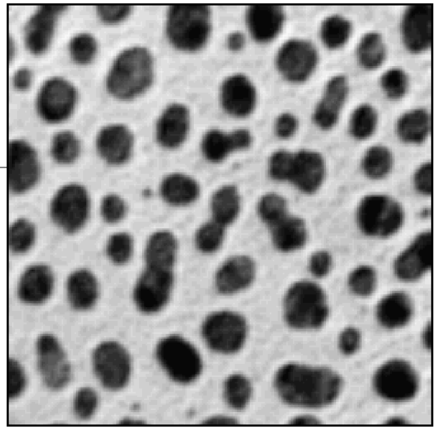
DILATION

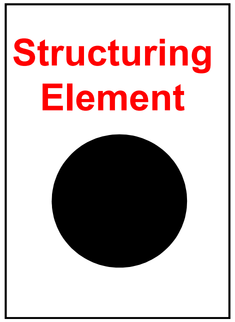


OPENING

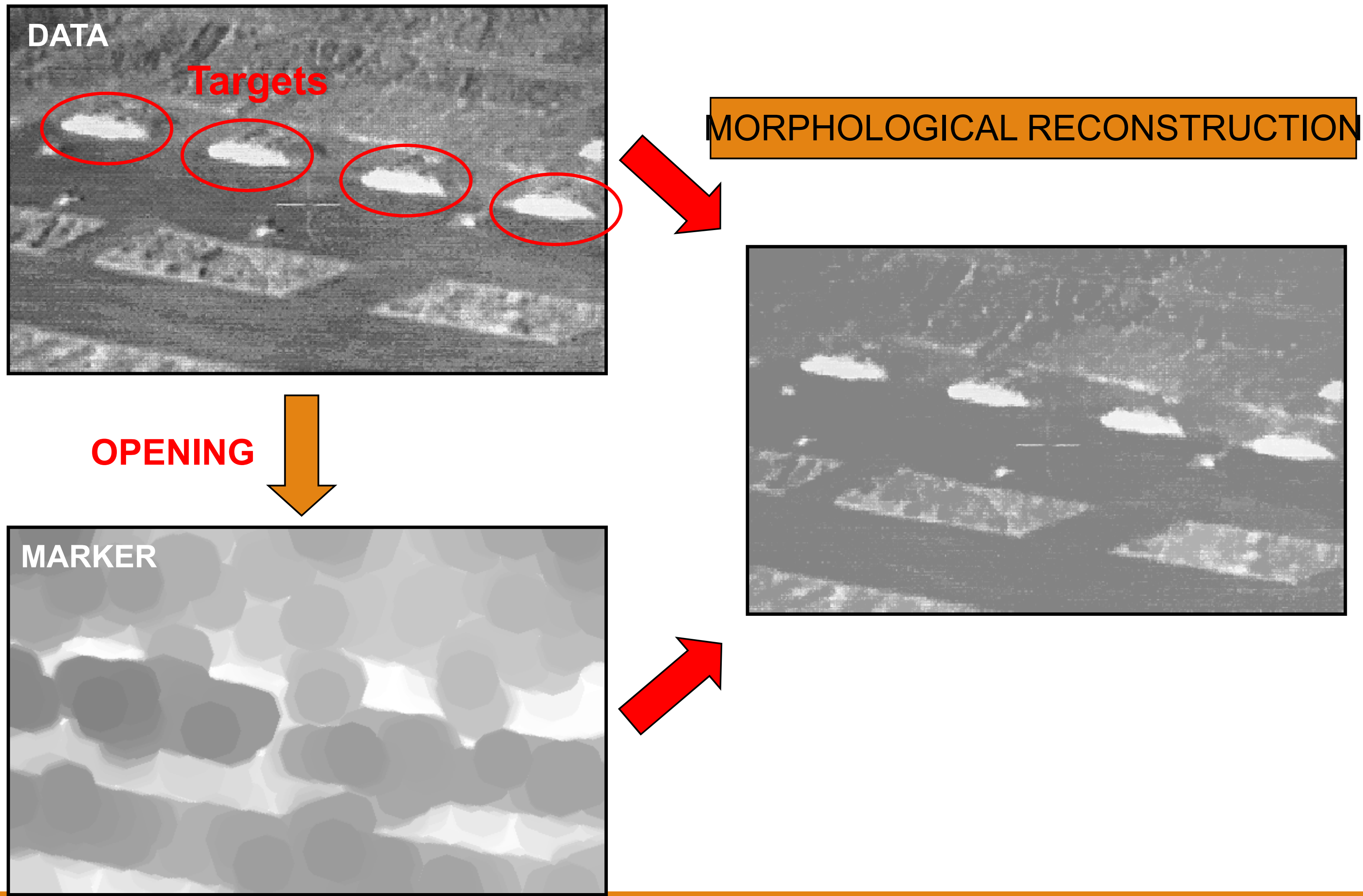
Gray scale opening



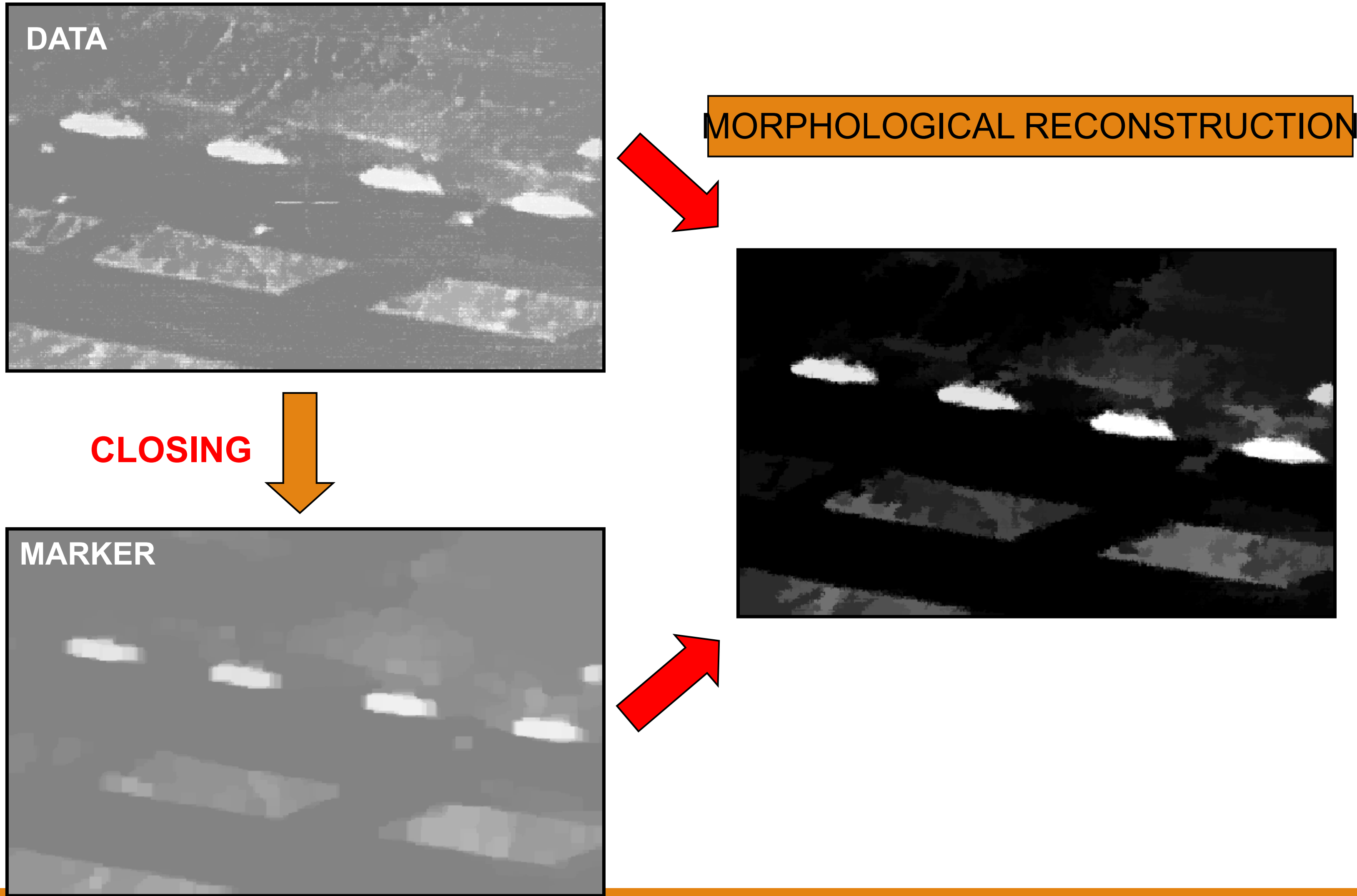




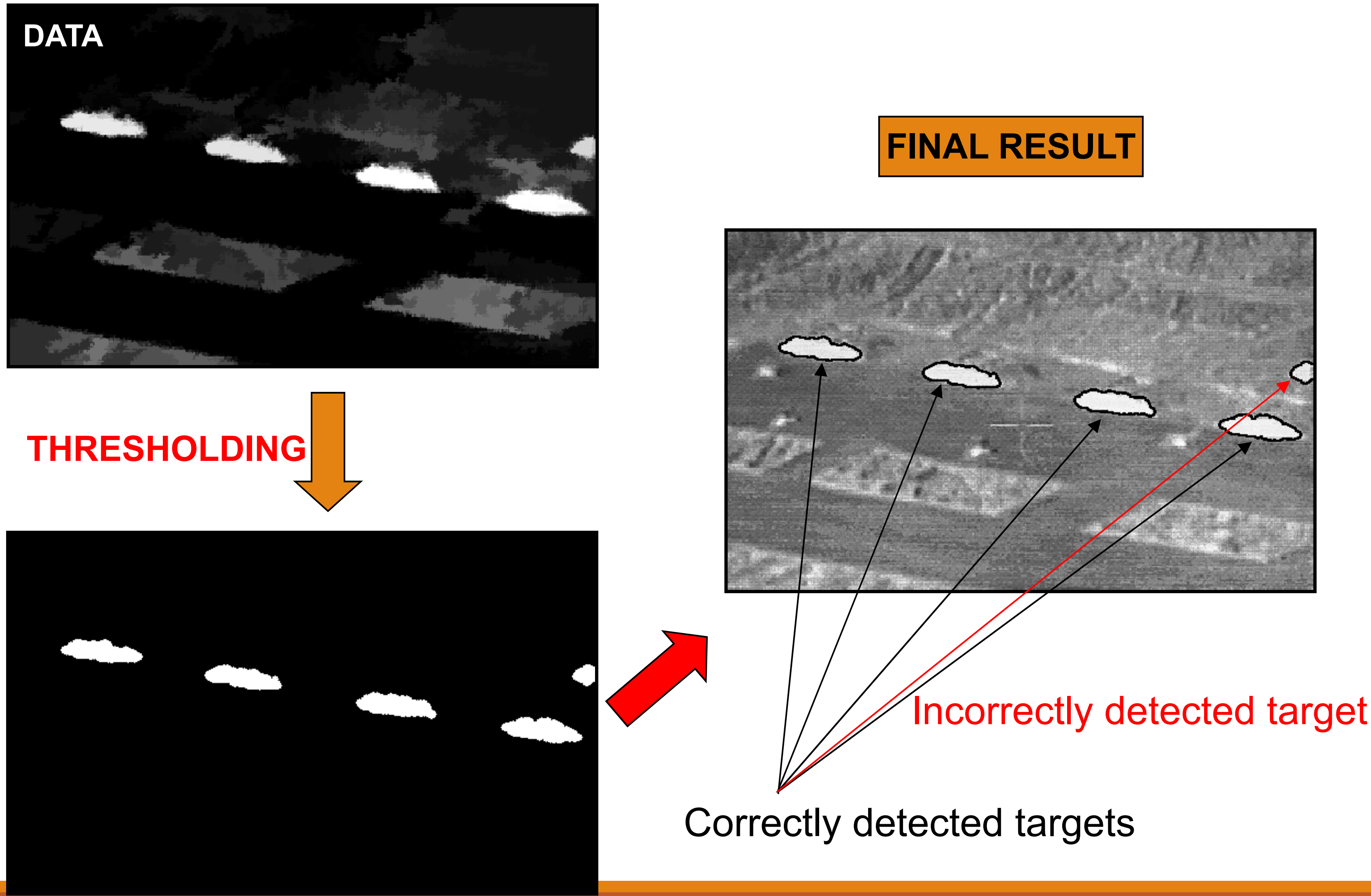
Target detection



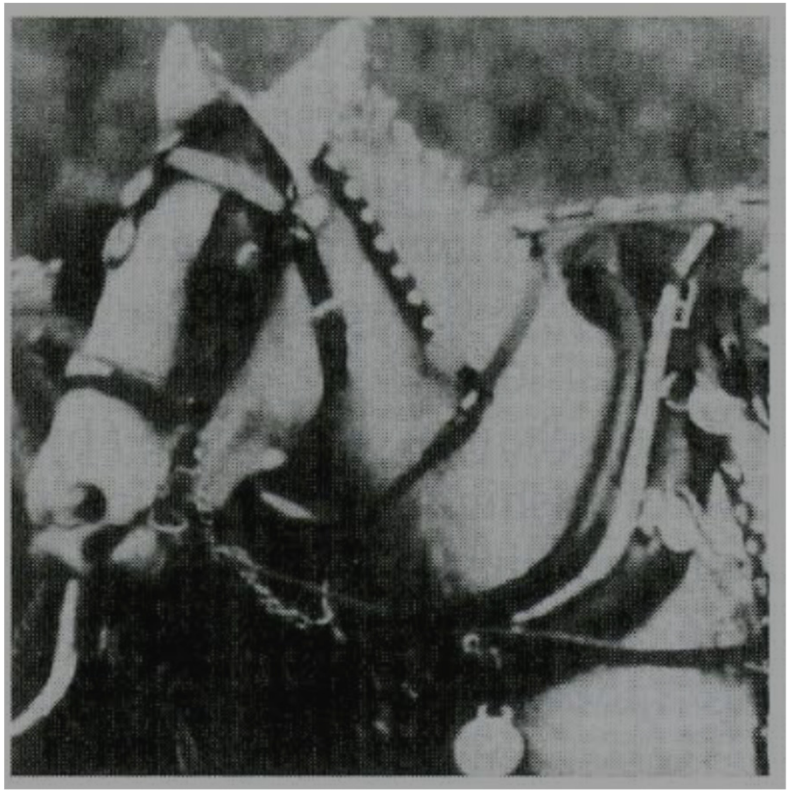
Application to Target Detection

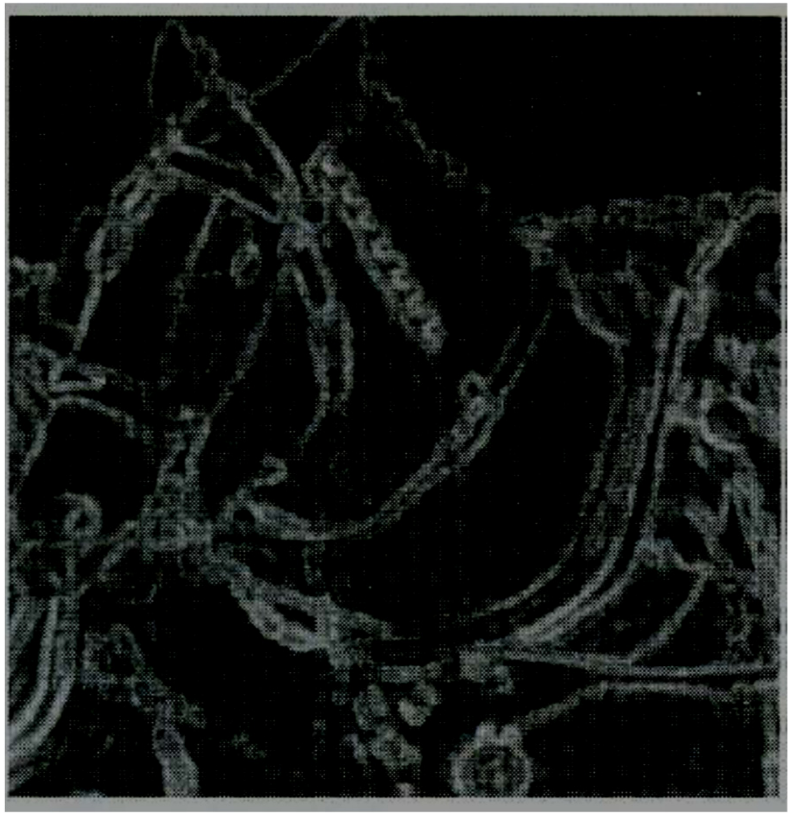


An Application: Target Detection



Morphological gradient

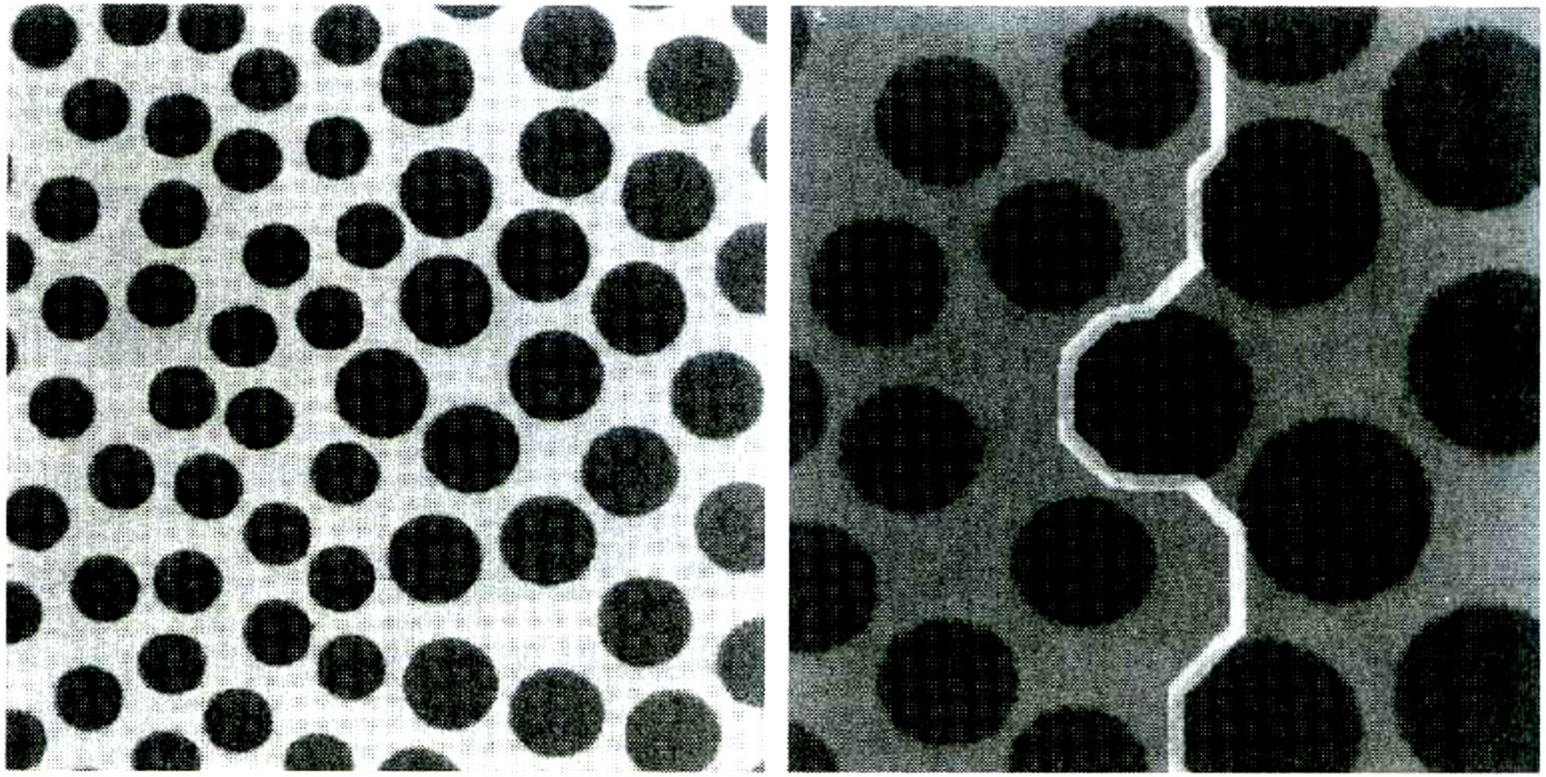




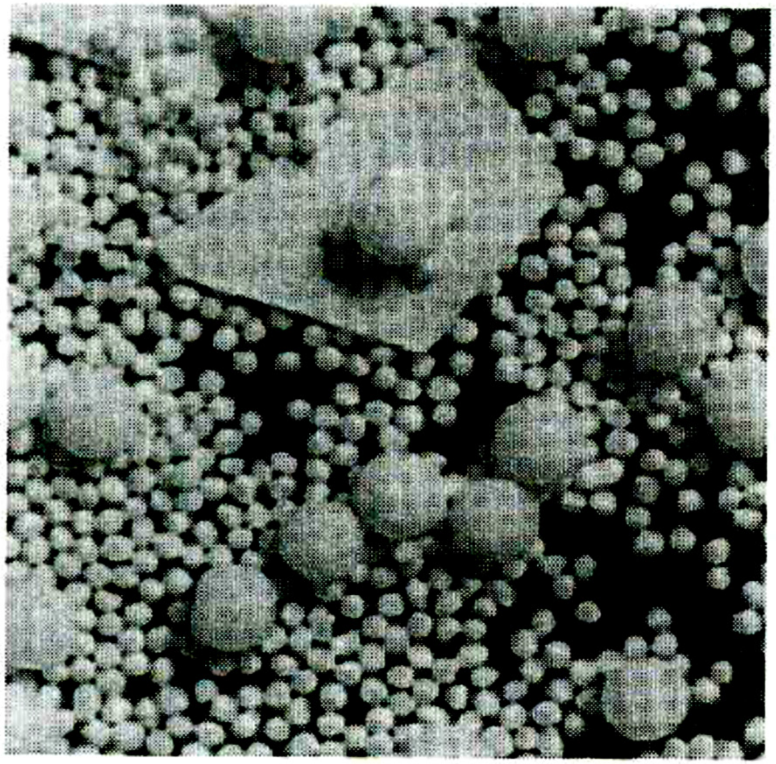
$$g = (f \oplus b) - (f \ominus b).$$

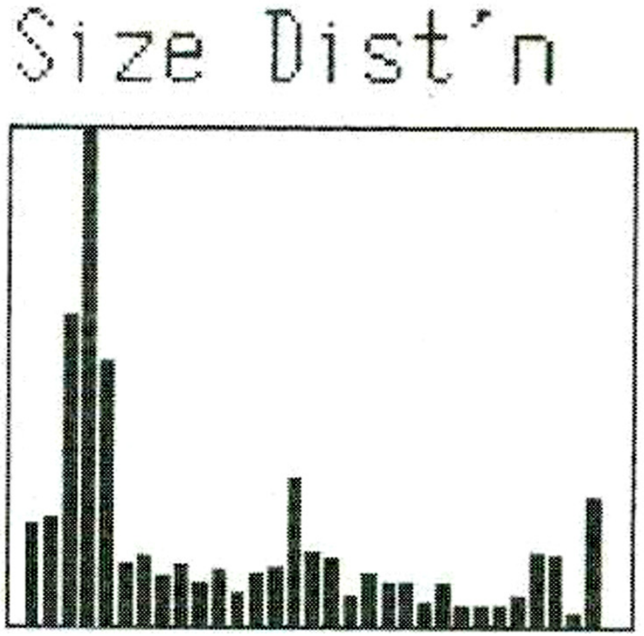
Very low sensibility to edge orientation

Texture segmentation

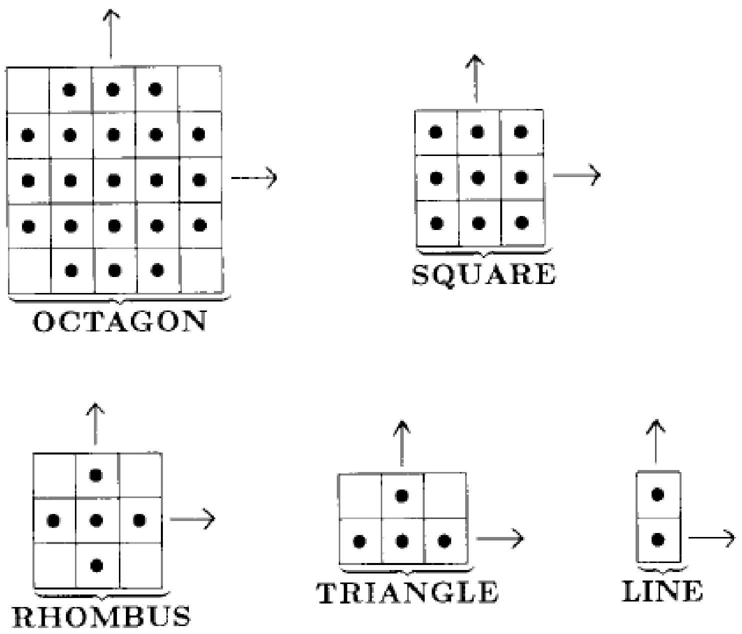


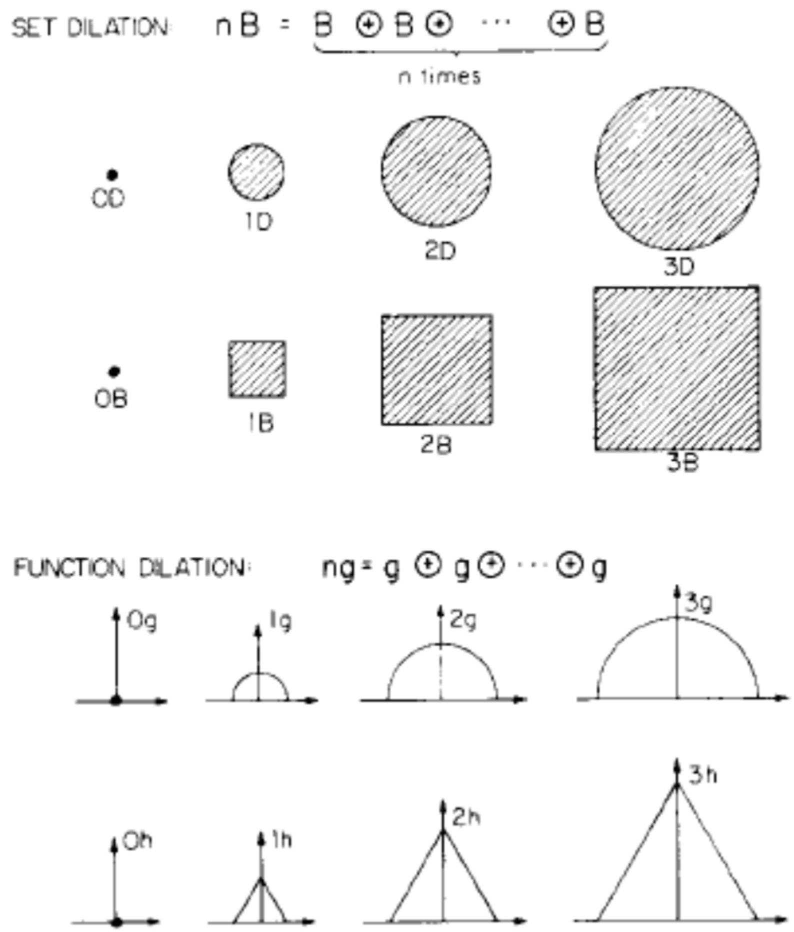
Granulometry



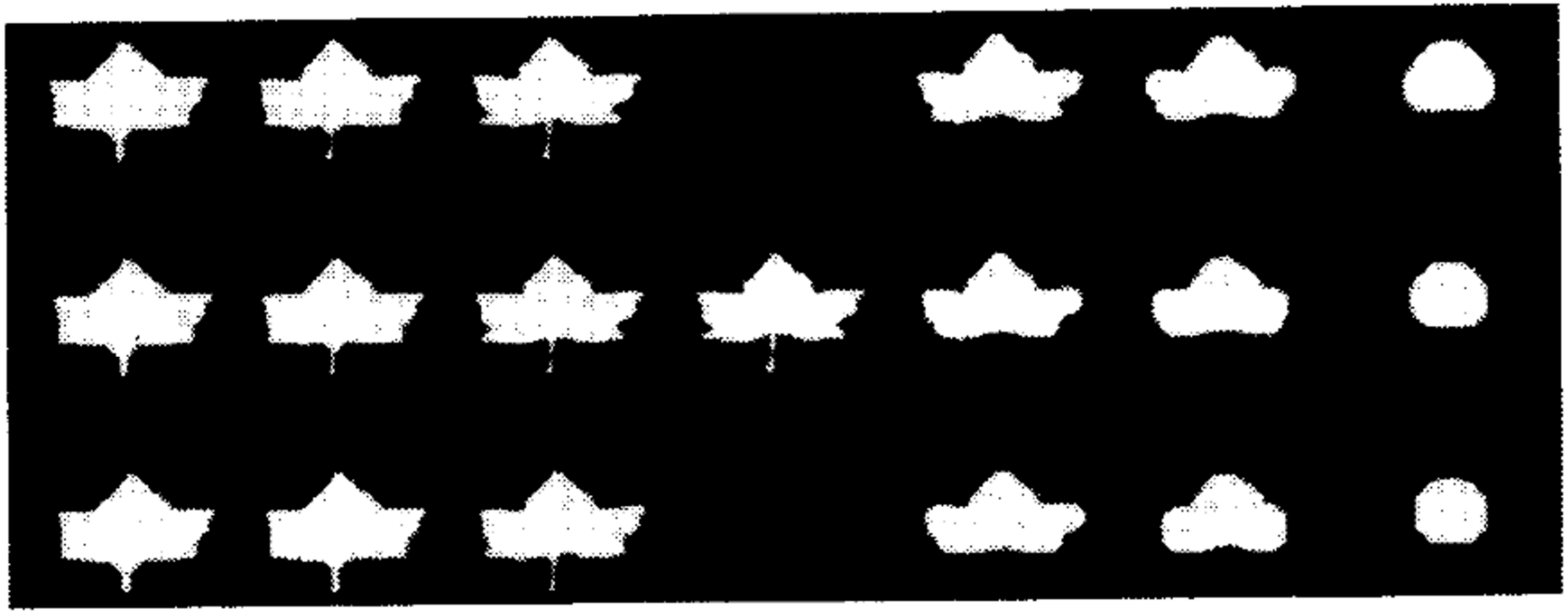


Pattern Spectrum (cenni)





Pattern Spectrum (cont.)

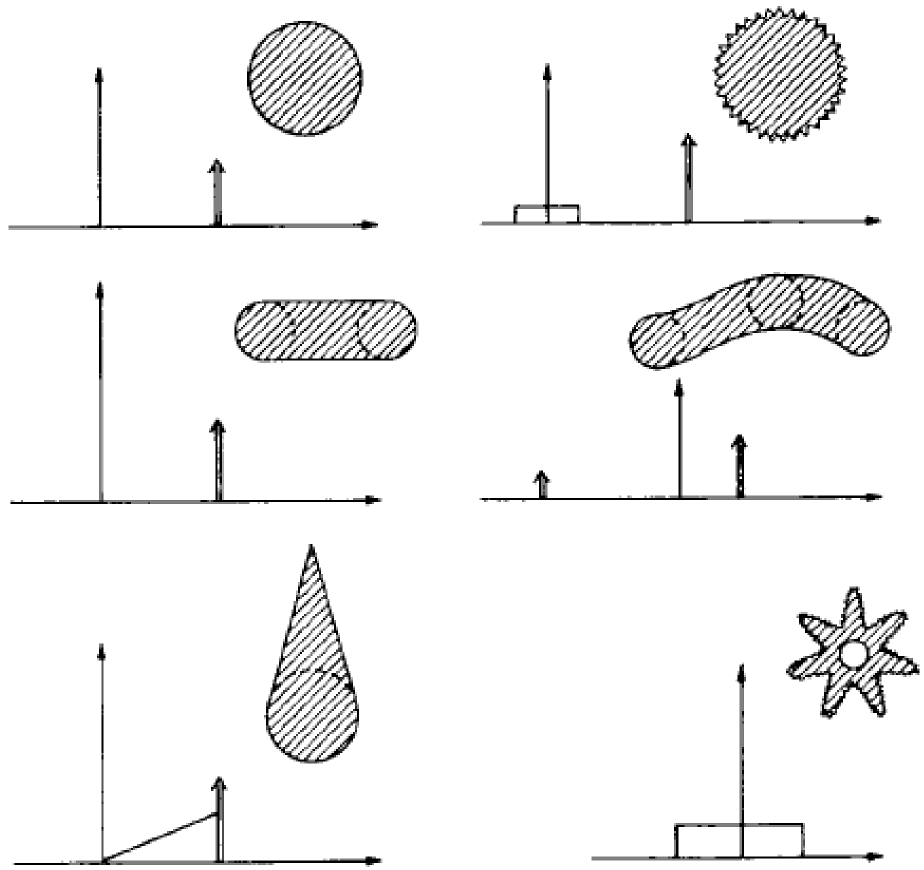


(g) (f) (e) (a) (b) (c) (d)

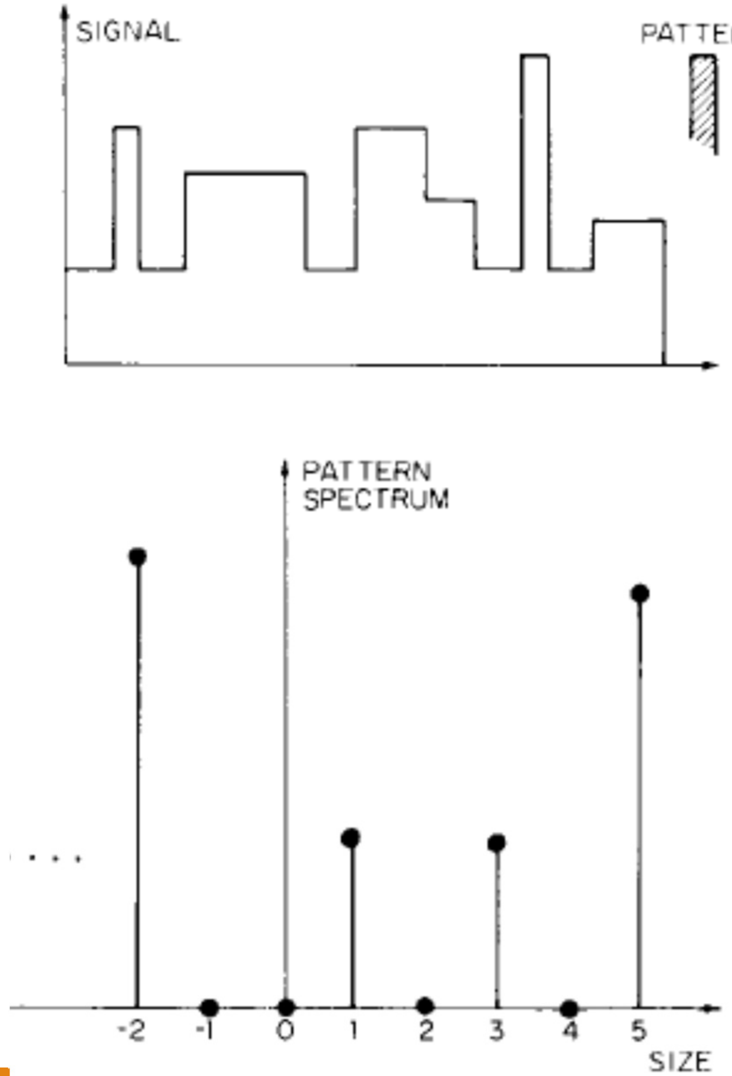
Fig. 3. Multiscale *SP* openings and closings. (a) Discrete binary image X of 85×128 pixels. (b) $X \circ nB$, $n = 1, 2, 3$ (top to bottom) where B is the octagon of Fig. 4. (c) $X \circ nB$, $n = 4, 5, 6$ (top to bottom). (d) $X \circ nB$, $n = 7, 8, 9$ (top to bottom). (e) $X \bullet nB$, $n = 1, 2, 3$ (top to bottom). (f) $X \bullet nB$, $n = 4, 5, 6$ (top to bottom). (g) $X \bullet nB$, $n = 7, 8, 9$ (top to bottom).

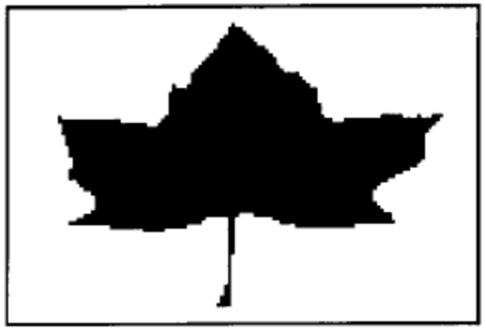
Pattern spectrum decomposition

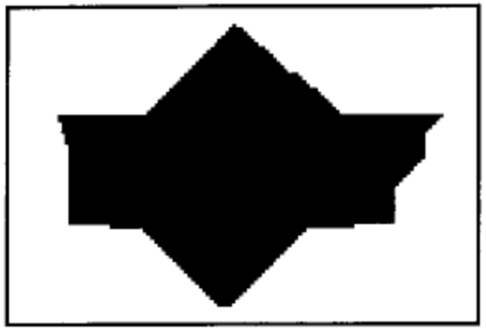
$$PS_X(r, B) = \frac{-dA(X \circ rB)}{dr},$$

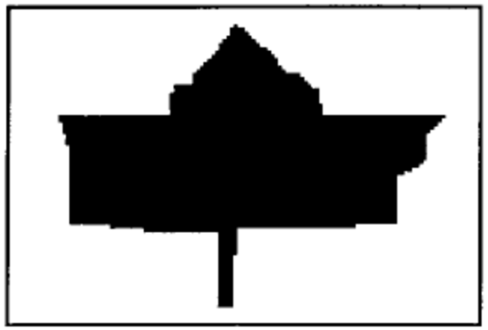


Trasformata Pattern Spectrum




(a)


(b)


(c)