Windowing

Lecture 6

Window method for FIR Design

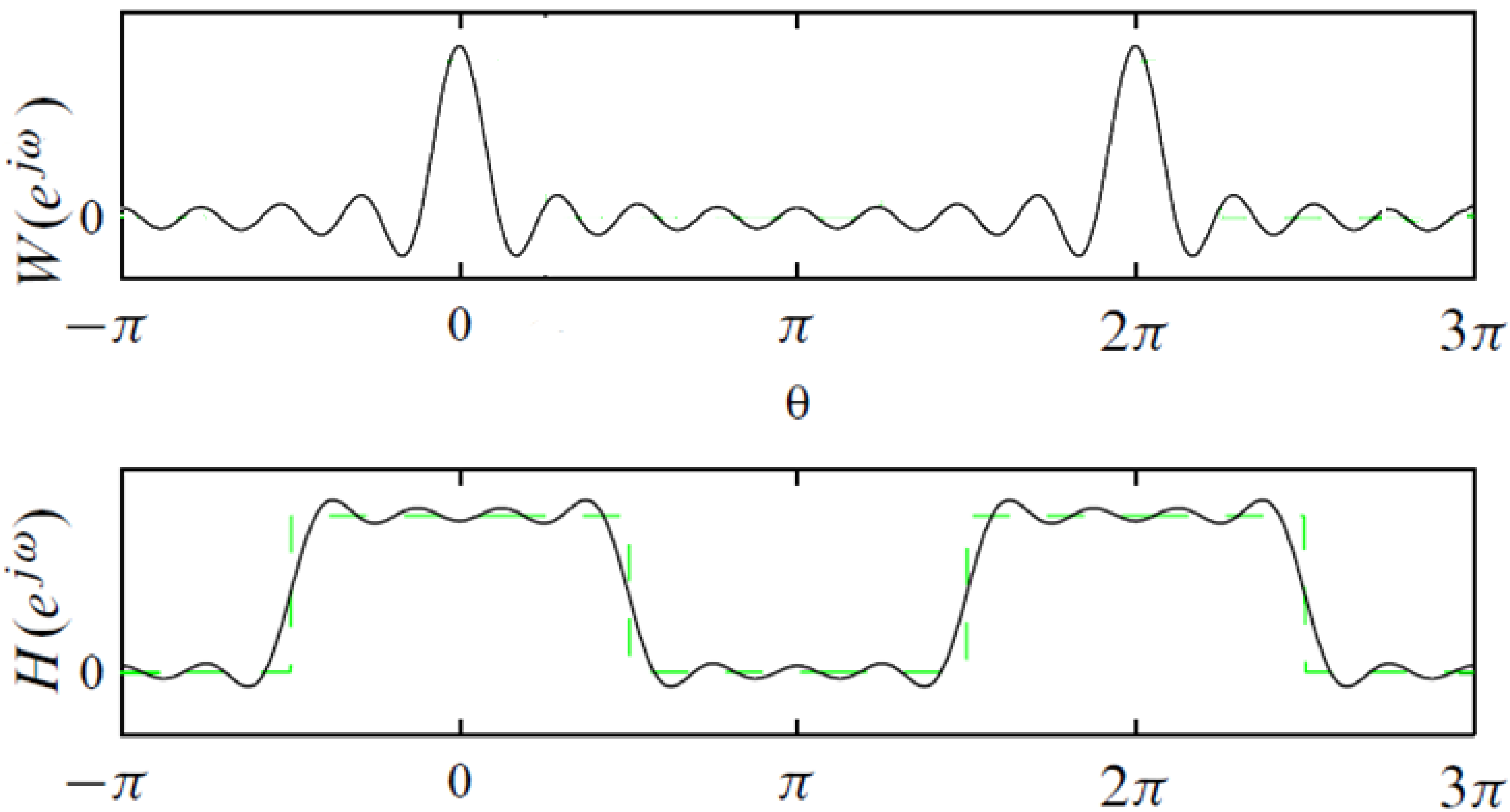
Assume that the desired filter response $H_d(e^{j\omega})$ is known. Using the inverse Fourier transform we can determine $h_d[n]$, the desired unit sample response. In the window method, a FIR filter is obtained by multiplying a window $w[n]$ with $h_d[n]$ to obtain a finite duration $h[n]$ of length N. This is required since $h_d[n]$ will in general be an infinite duration sequence, and the corresponding filter will therefore not be realisable. If $h_d[n]$ is even or odd symmetric and $w[n]$ is even symmetric, then $h_d[n]w[n]$ is a linear phase filter.

Two important design criteria are the *length* and *shape* of the window $w[n]$. To see how these factors influence the design, consider the multiplication operation in the frequency domain: since $h[n] = h_d[n]w[n]$,

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega}).$$

Window method for FIR

The following plot demonstrates the convolution operation. In each case the dotted line indicates the desired response $H_d(e^{j\omega})$.



Window method

From this, note that

- The *mainlobe* width of $W(e^{j\omega})$ affects the *transition* width of $H(e^{j\omega})$. Increasing the length N of $h[n]$ reduces the mainlobe width and hence the transition width of the overall response.
- The *sidelobes* of $W(e^{j\omega})$ affect the passband and stopband tolerance of $H(e^{j\omega})$. This can be controlled by changing the shape of the window. Changing N does not affect the sidelobe behaviour.

Common windows

Some commonly used windows for filter design are

- **Rectangular:**

$$w[n] = \begin{cases} 1 & 0 \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

- **Bartlett (triangular):**

$$w[n] = \begin{cases} 2n/N & 0 \leq n \leq N/2 \\ 2 - 2n/N & N/2 < n \leq N \\ 0 & \text{otherwise} \end{cases}$$

Common windows

- **Hanning:**

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/N) & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

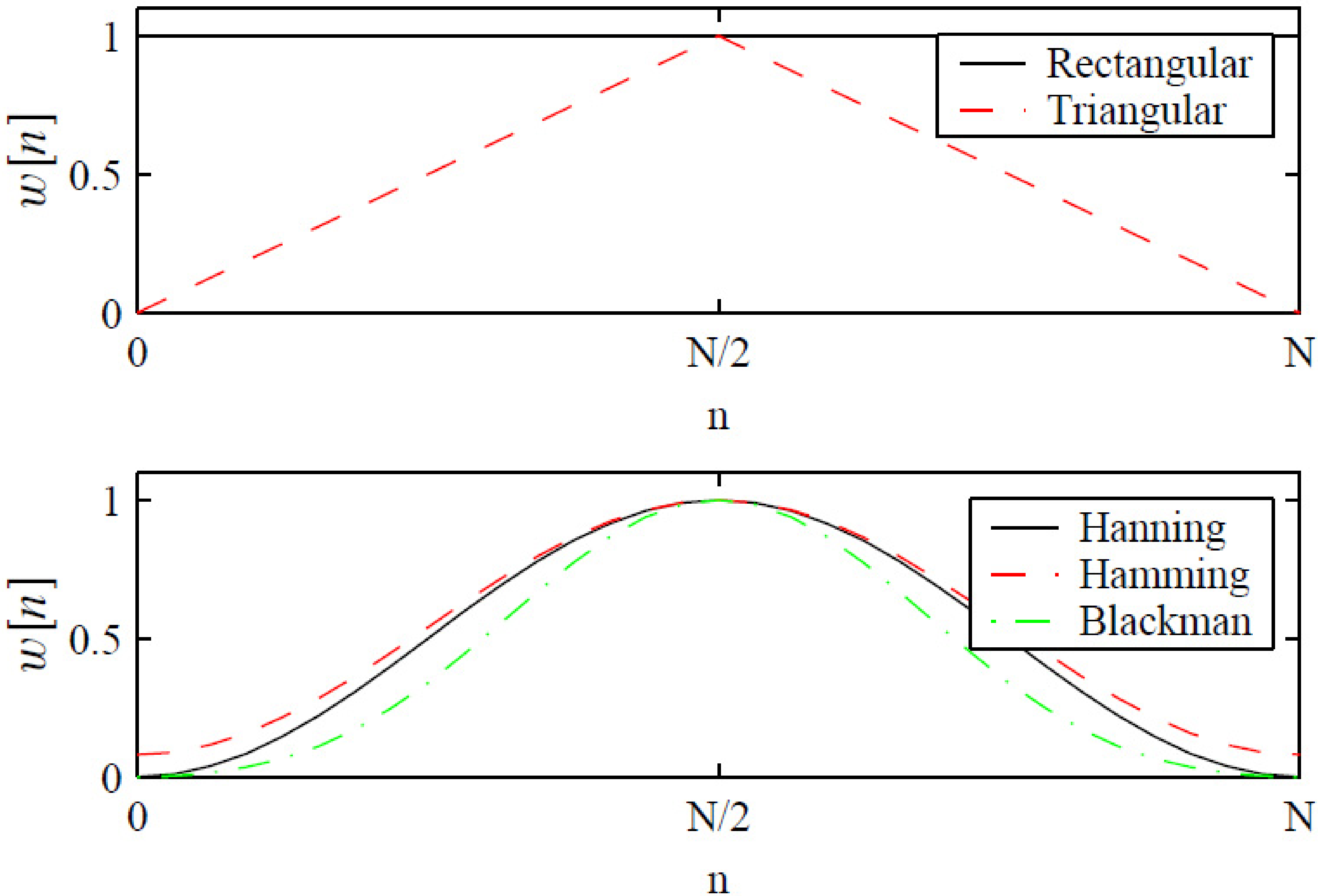
- **Hamming:**

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/N) & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

- **Kaiser:**

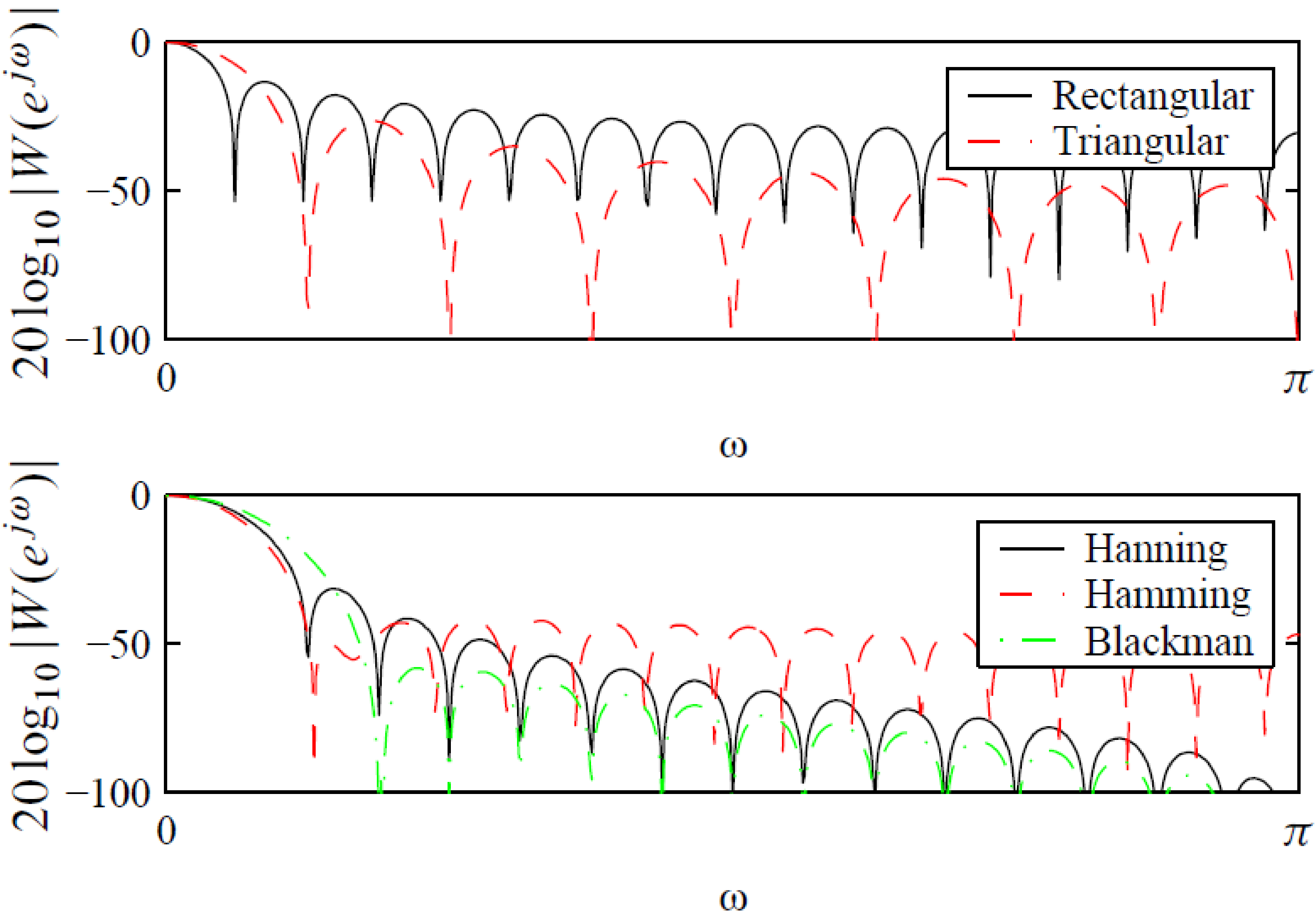
$$w[n] = \begin{cases} I_0[\beta(1 - [(n-\alpha)/\alpha]^2)^{1/2}] & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

Examples of these windows



Windows frequency response

All windows trade off a reduction in sidelobe level against an increase in mainlobe width. This is demonstrated below in a plot of the frequency response of each of the windows:



Important windows characteristics

Window	Peak sidelobe amplitude (dB)	Mainlobe transition width	Peak approximation error (dB)
Rectangular	-13	$4\pi/(N+1)$	-21
Bartlett	-25	$8\pi/N$	-25
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53

The Kaiser window has a number of parameters that can be used to explicitly tune the characteristics.

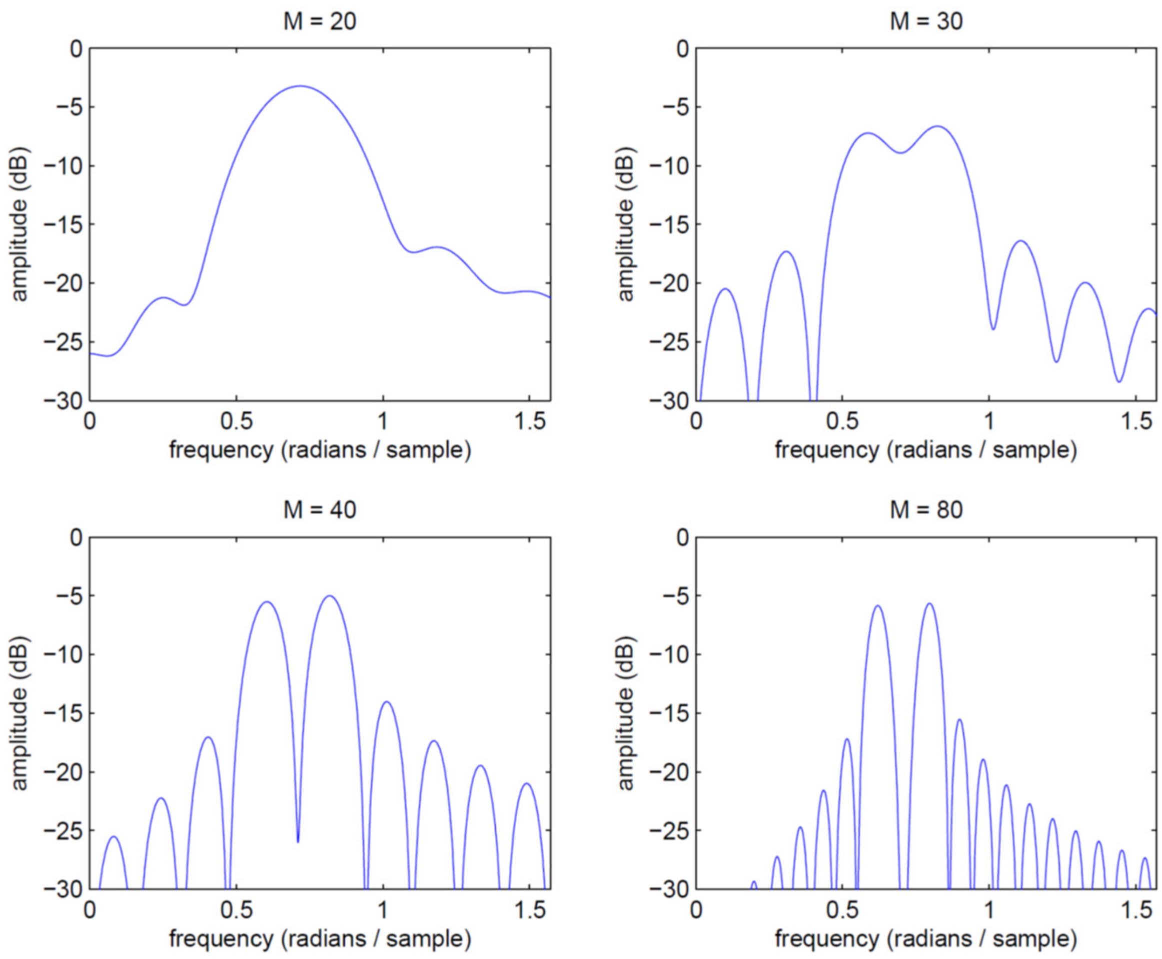
In practice, the window shape is chosen first based on passband and stopband tolerance requirements. The window size is then determined based on transition width requirements. To determine $h_d[n]$ from $H_d(e^{j\omega})$ one can sample $H_d(e^{j\omega})$ closely and use a large inverse DFT.

resolution of two cosines

- 2 cosines separated by $\Delta\omega = \frac{2\pi}{40}$
- rectangular windows of lengths: 20, 30, 40, 80:

$$\Delta\omega = \frac{1}{2}\Omega_M, \frac{3}{4}\Omega_M, \Omega_M, 2\Omega_M$$

In phase example



Phase quadrature case