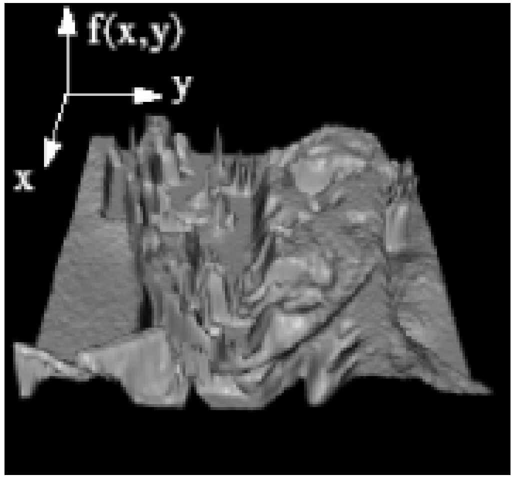
From 1D to 2D Signals

Lecture 9

What is an image?

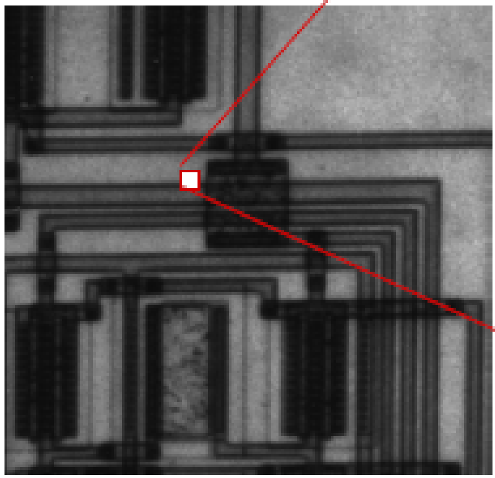
- Ideally, we think of an **image** as a **2-dimensional light intensity function**, $f(x,y)$, where x and y are spatial coordinates, and f at (x,y) is related to the brightness or color of the image at that point.
- In practice, most images are defined over a rectangle.
- Continuous in amplitude („continuous-tone“)
- Continuous in space: no pixels!



Digital Images and Pixels

- A **digital image is the representation of a continuous image** *f(x,y) by a 2-d array of discrete samples. The amplitude of* each sample is quantized to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a **pixel** or **pel (from "picture element")**
- Pixels are point samples, without extent.
- A pixel is not:
  - Round, square, or rectangular
  - An element of an image sensor
  - m An element of a display

A Digital Image is Represented by Numbers



272 pixels

280 pixels

128	125	107	105	110	118	116	114	110
121	122	115	108	106	107	116	116	107
110	114	112	107	105	103	106	106	100
100	96	100	99	94	94	101	101	89
85	82	81	80	76	75	80	82	72
58	58	56	54	53	52	51	49	45
41	41	41	39	39	38	36	35	33
43	43	42	43	41	41	41	43	40
60	60	59	59	60	59	59	58	56

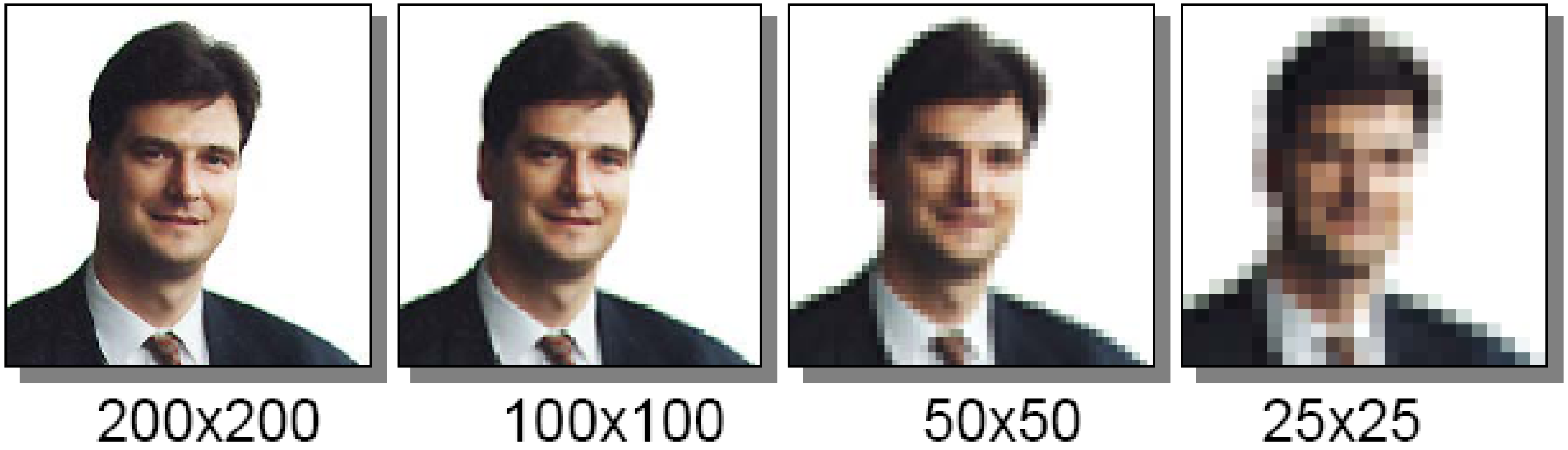
- Pixel = "picture element"
- Represents brightness at one point

An image can be represented as a matrix

- The pixel values *f(x,y) are sorted into the matrix in "natural" order, with x corresponding to the column and y to the row index.*
- *Matlab, instead, uses matrix* convention. This results in $f(x,y) = f_{yx}$, *where* f_{yx} *denotes an individual* element in common matrix notation.
- For a color image, **f** might be one of the components.

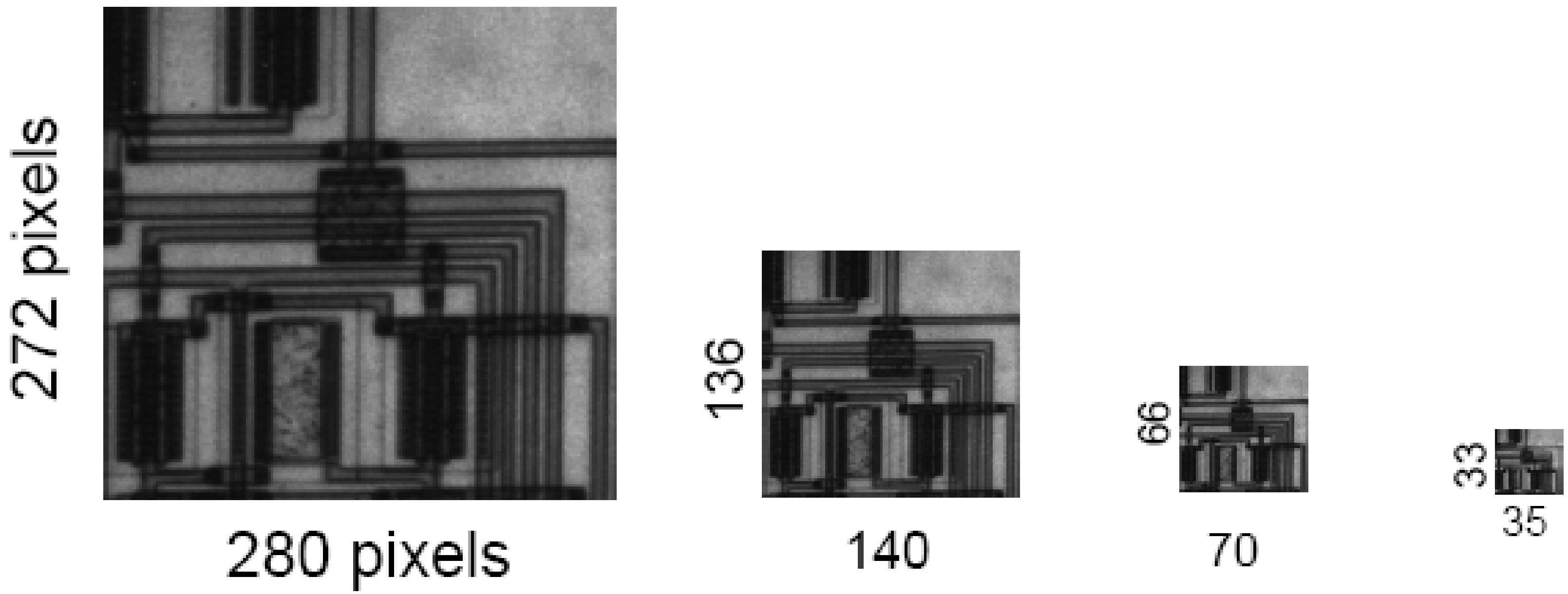
$$\mathbf{f} = \begin{bmatrix} f(0,0) & f(1,0) & \cdots & f(N-1,0) \\ f(0,1) & f(1,1) & \cdots & f(N-1,1) \\ \vdots & \vdots & & \vdots \\ f(0,L-1) & f(1,L-1) & \cdots & f(N-1,L-1) \end{bmatrix} \quad \xrightarrow{x} \quad \downarrow y$$

Image Size and Resolution

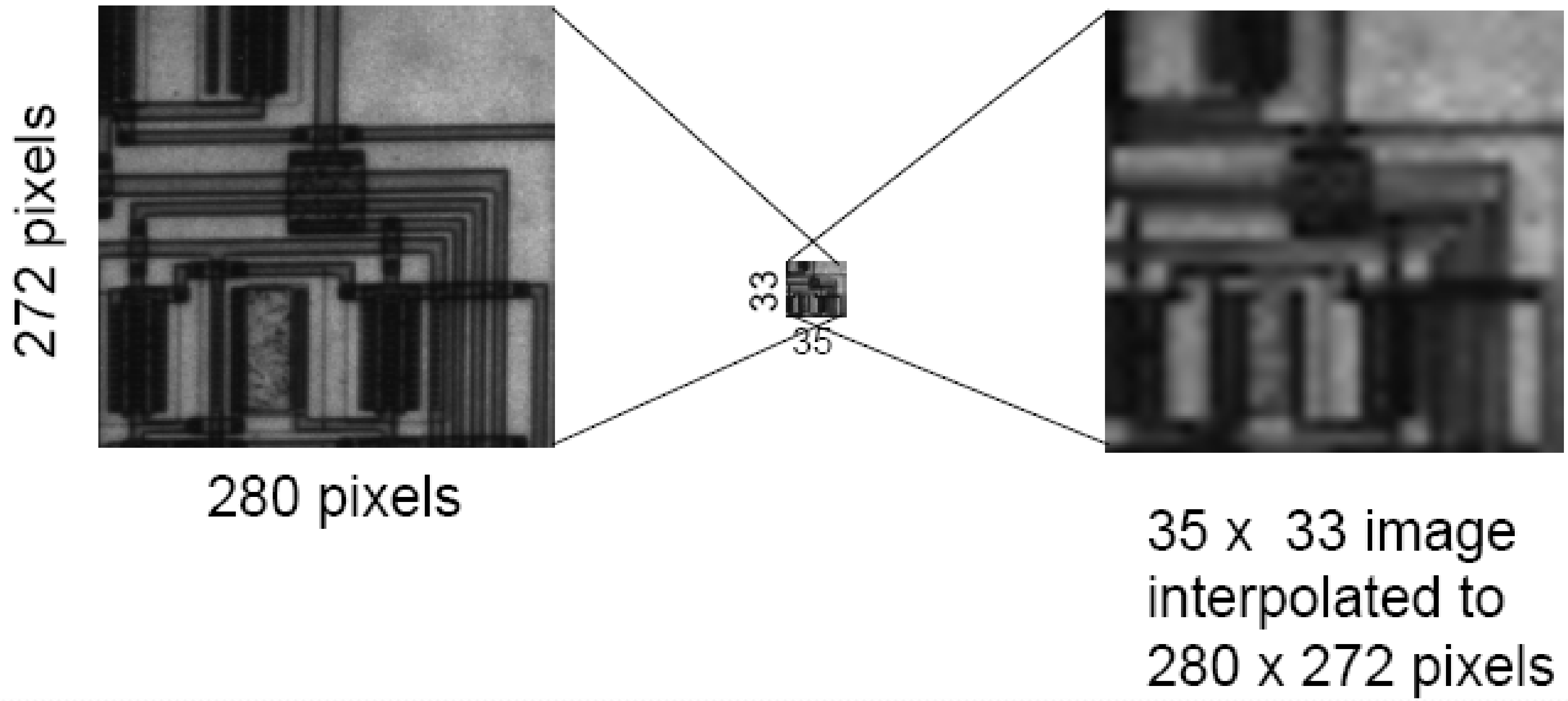


- These images were produced by simply picking every *n-th* sample horizontally and vertically and replicating that value *nxn* times.
- We can do better
  - *prefiltering before subsampling to avoid aliasing*
  - *Smooth interpolation*

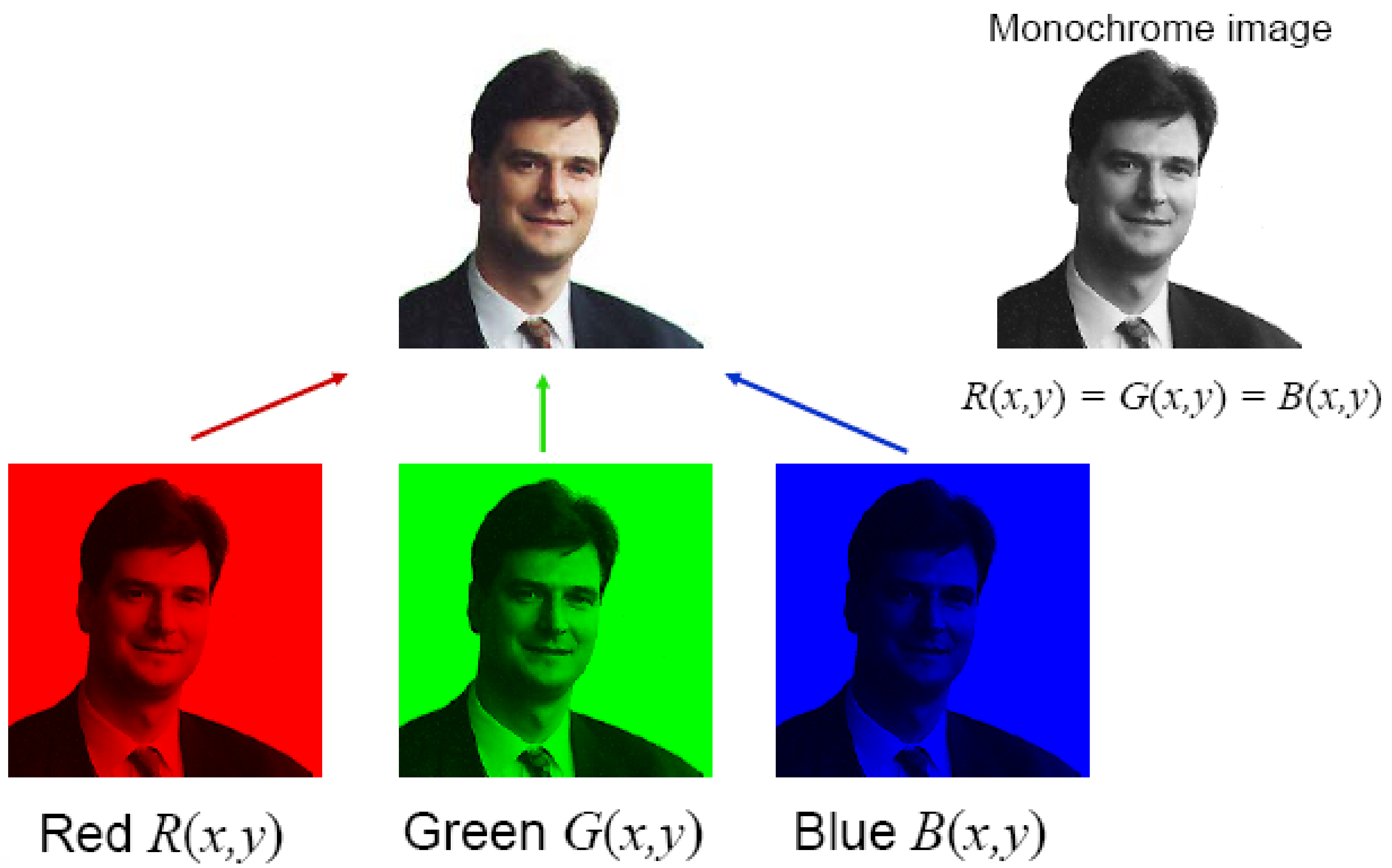
Image of different size



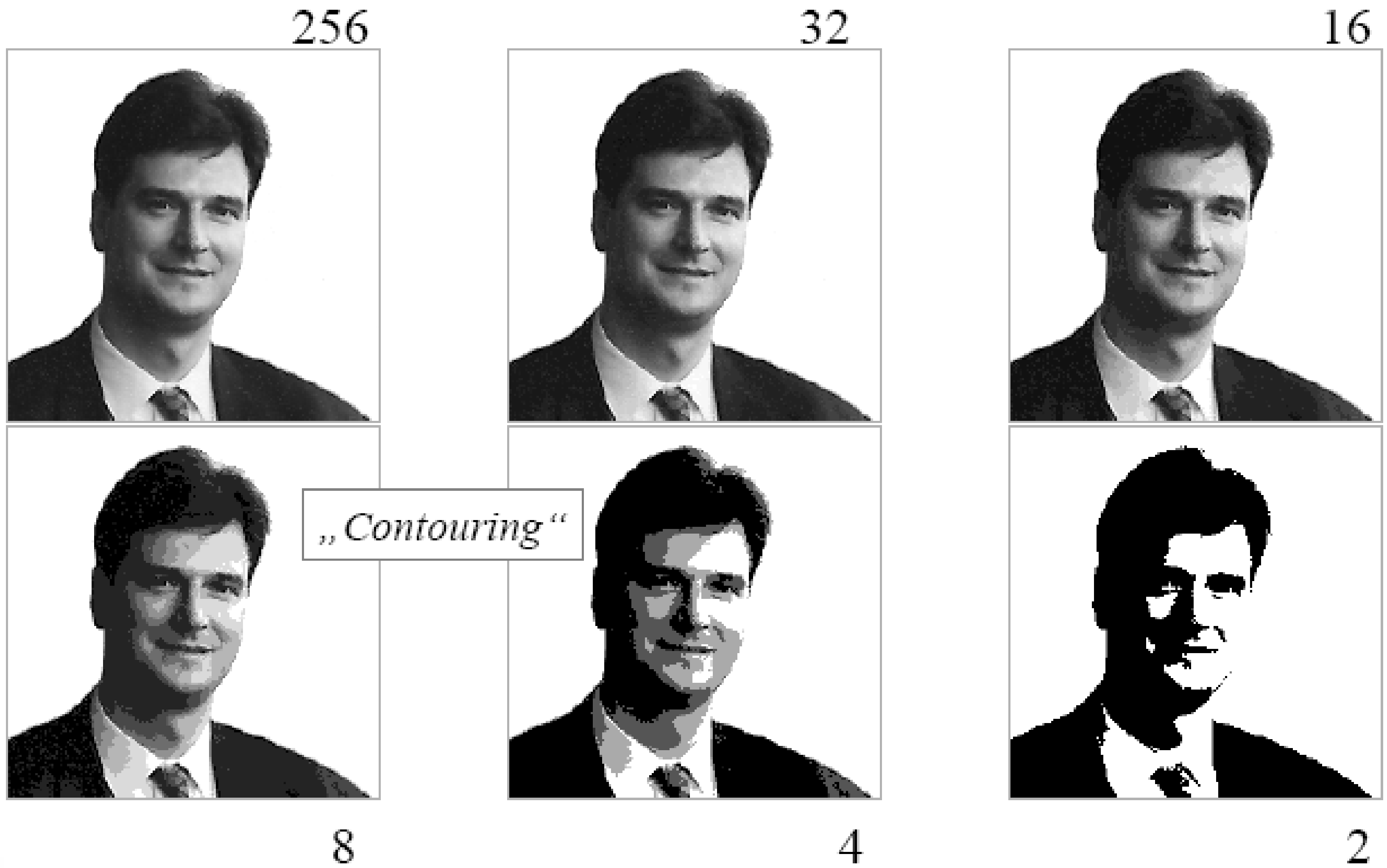
Fewer Pixels Mean Lower Spatial Resolution



Color Components

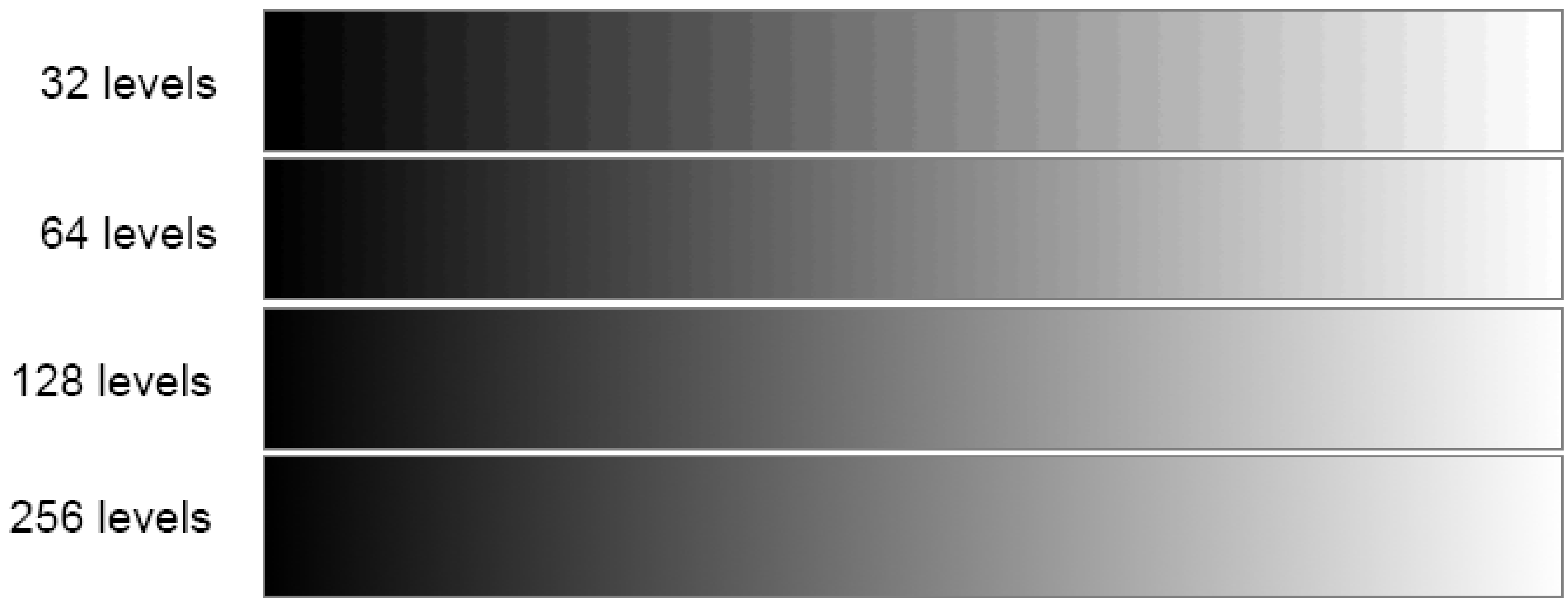


Different numbers of gray levels



How many gray levels are required?

- How many gray levels are required?



- Digital images typically are quantized to 256 gray levels.

Storage requirements for digital images

- Image LxN pixels, 2^B *gray levels, c color components*

  Size = LxNxBxc

  - *Example: L=N=512, B=8, c=1 (i.e., monochrome) Size = 2,097,152 bits (or 256 kByte)*
  - *Example: LxN=1024x1280, B=8, c=3 (24 bit RGB image) Size = 31,457,280 bits (or 3.75 MByte)*
- Much less with (lossy) compression!
- For a video multiply by the frame rate and by the number of seconds of its lenght.

Histograms

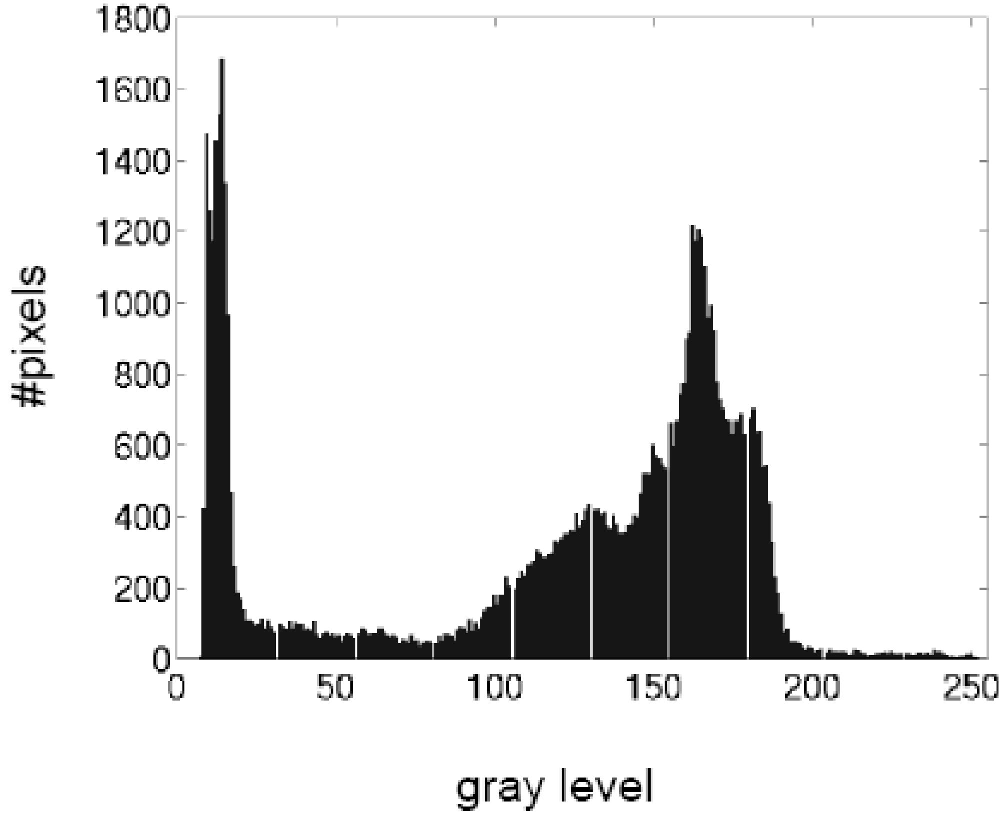
- Distribution of gray-levels can be judged by measuring a histogram:
  - For B-bit image, initialize 2^B counters with 0
  - Loop over all pixels *x,y*

- When encountering gray level *f(x,y)=i, increment counter #i*

- Histogram can be interpreted as an estimate of the probability density function (*pdf*) of an underlying random process.

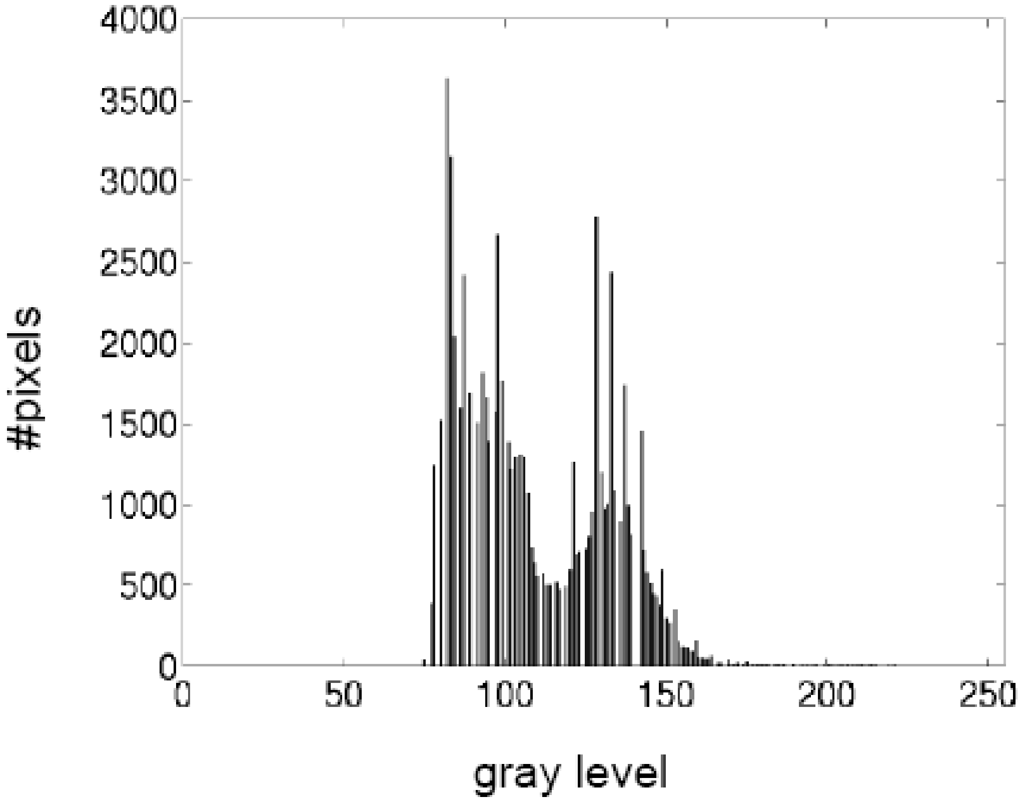
- You can also use fewer, larger bins to trade off amplitude resolution against sample size.

Example for the histogram



Cameraman
image

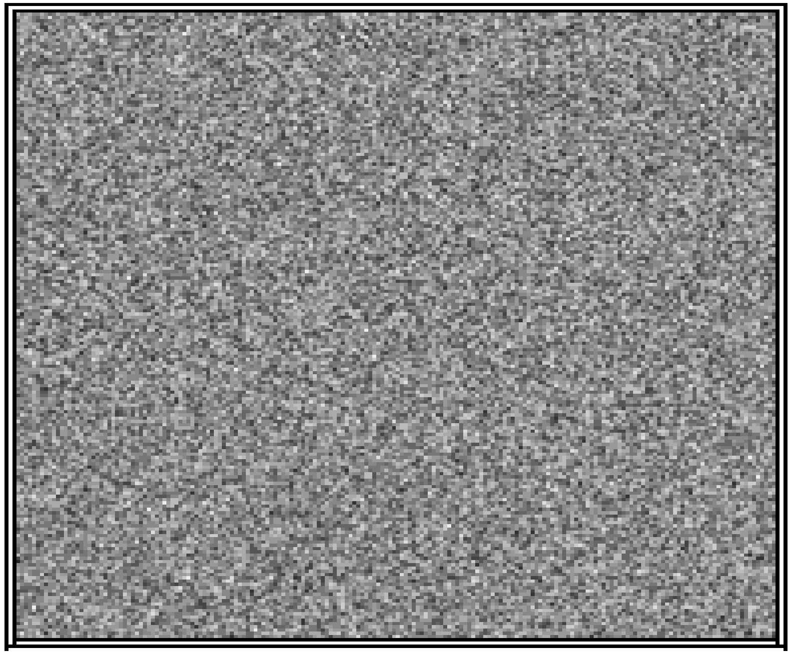
Histogram Example

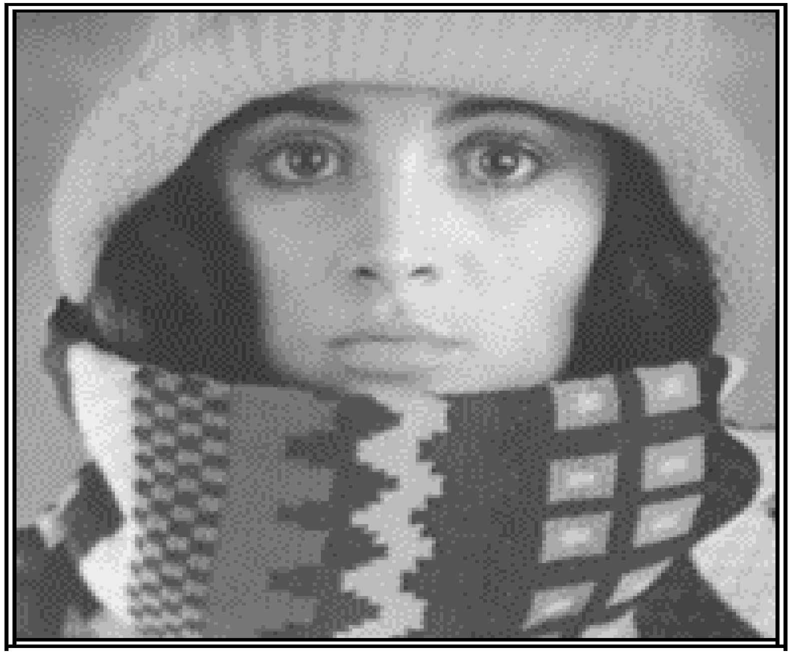


Pout image

Histogram comparison

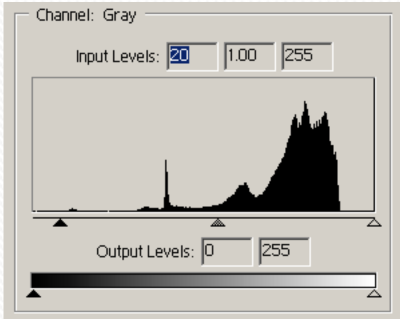
- Both these images present the same Histogram

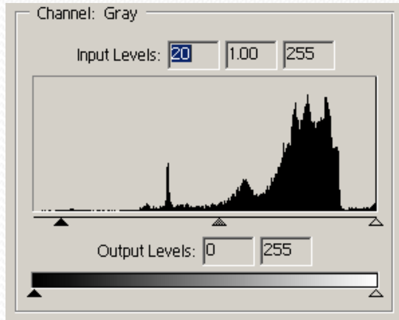


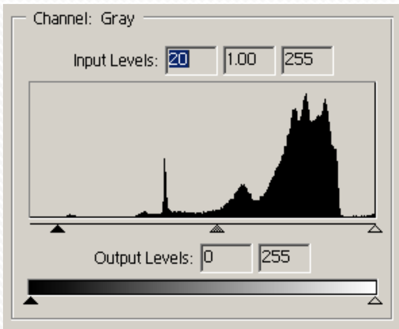


Histogram comparison

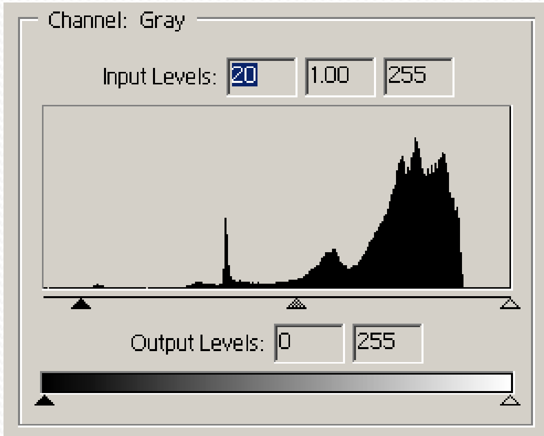
- Histogram as an invariant feature







Histogram comparison



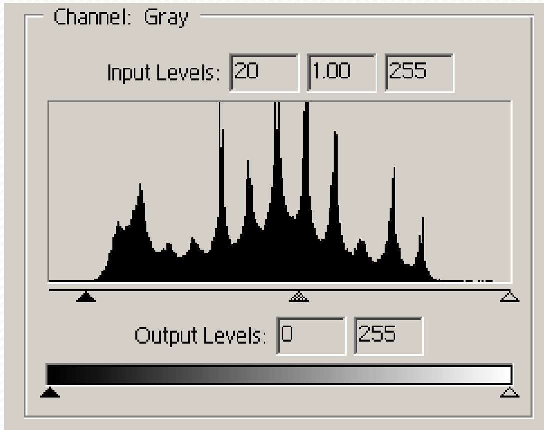
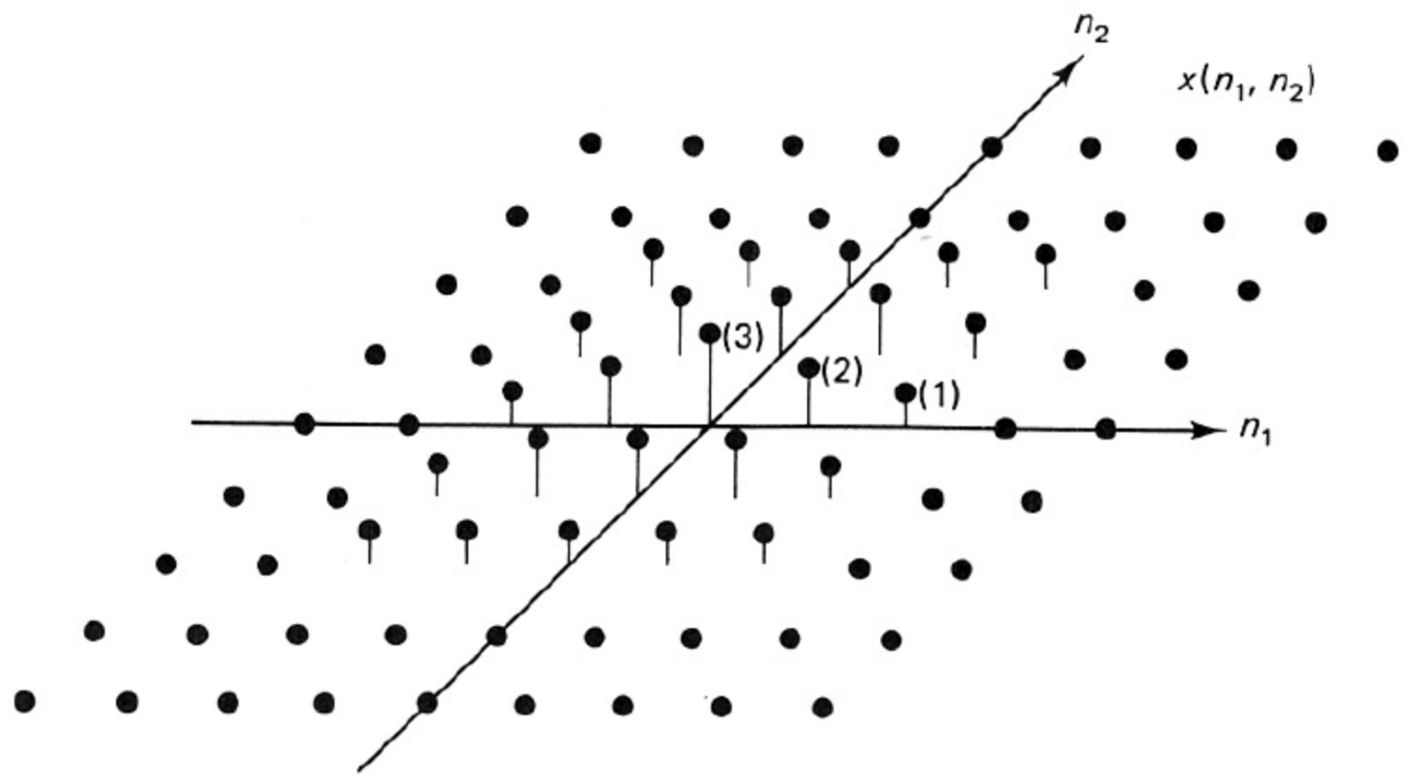


Image as a 2D sampling

- A digital image can be considered as a 2D discrete signal

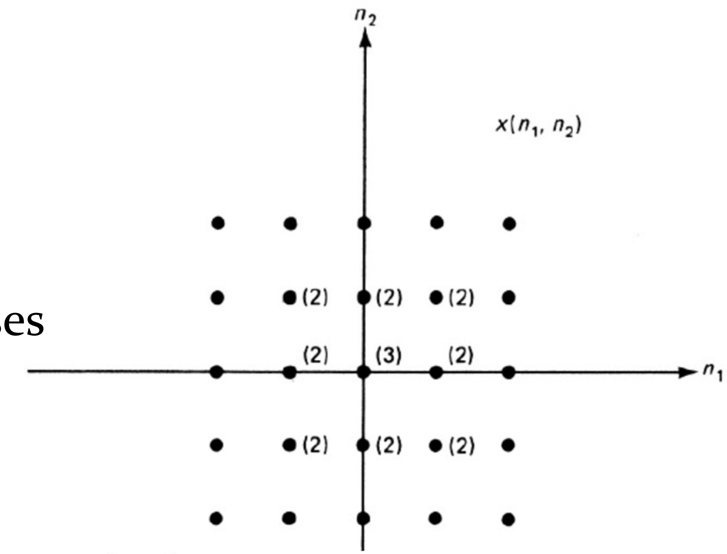
$$x(n_1, n_2)$$



Impulse definition

$$\delta(n_1, n_2) = \begin{cases} 1, & n_1 = n_2 = 0 \\ 0, & \text{otherwise.} \end{cases}$$

Each sequence can be considered as a sequence of impulses



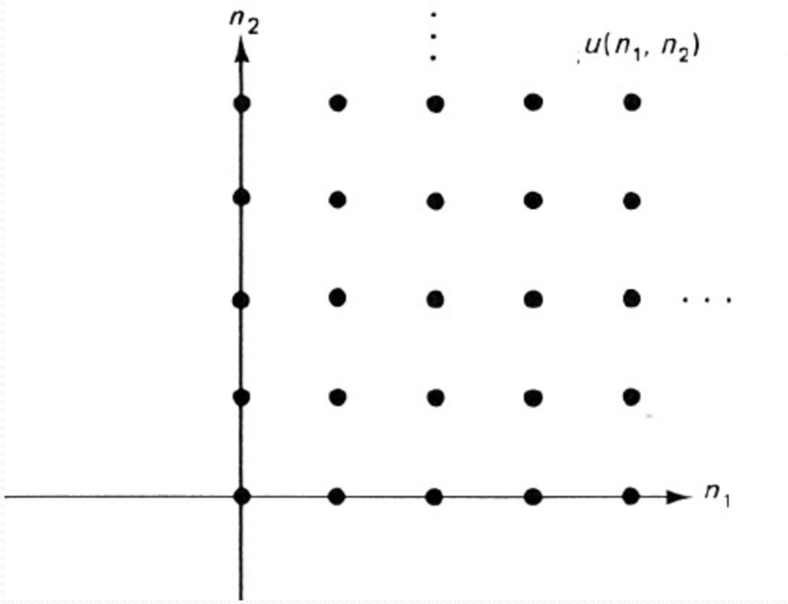
$$\begin{aligned} x(n_1,n_2) = \cdots &+ x(-1,-1)\,\delta(n_1+1,n_2+1) + x(0,-1)\,\delta(n_1,n_2+1) \\ &+ x(1,-1)\,\delta(n_1-1,n_2+1) + \cdots + x(-1,0)\,\delta(n_1+1,n_2) \\ &+ x(0,0)\,\delta(n_1,n_2) + x(1,0)\,\delta(n_1-1,n_2) \\ &+ \cdots + x(-1,1)\,\delta(n_1+1,n_2-1) \\ &+ x(0,1)\,\delta(n_1,n_2-1) + x(1,1)\,\delta(n_1-1,n_2-1) + \cdots \\ = &\sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1,k_2)\,\delta(n_1-k_1,n_2-k_2). \end{aligned}$$

The step function

- The step function a combination of impulses.

$$u(n_1, n_2) = \sum_{k_1=-\infty}^{n_1} \sum_{k_2=-\infty}^{n_2} \delta(k_1, k_2)$$

$$\delta(n_1, n_2) = u(n_1, n_2) - u(n_1 - 1, n_2) - u(n_1, n_2 - 1) + u(n_1 - 1, n_2 - 1).$$



Separable sequences

- A separable 2D sequence can be written as:

$$x(n_1, n_2) = f(n_1)g(n_2)$$

- The impulse and step function are separable functions

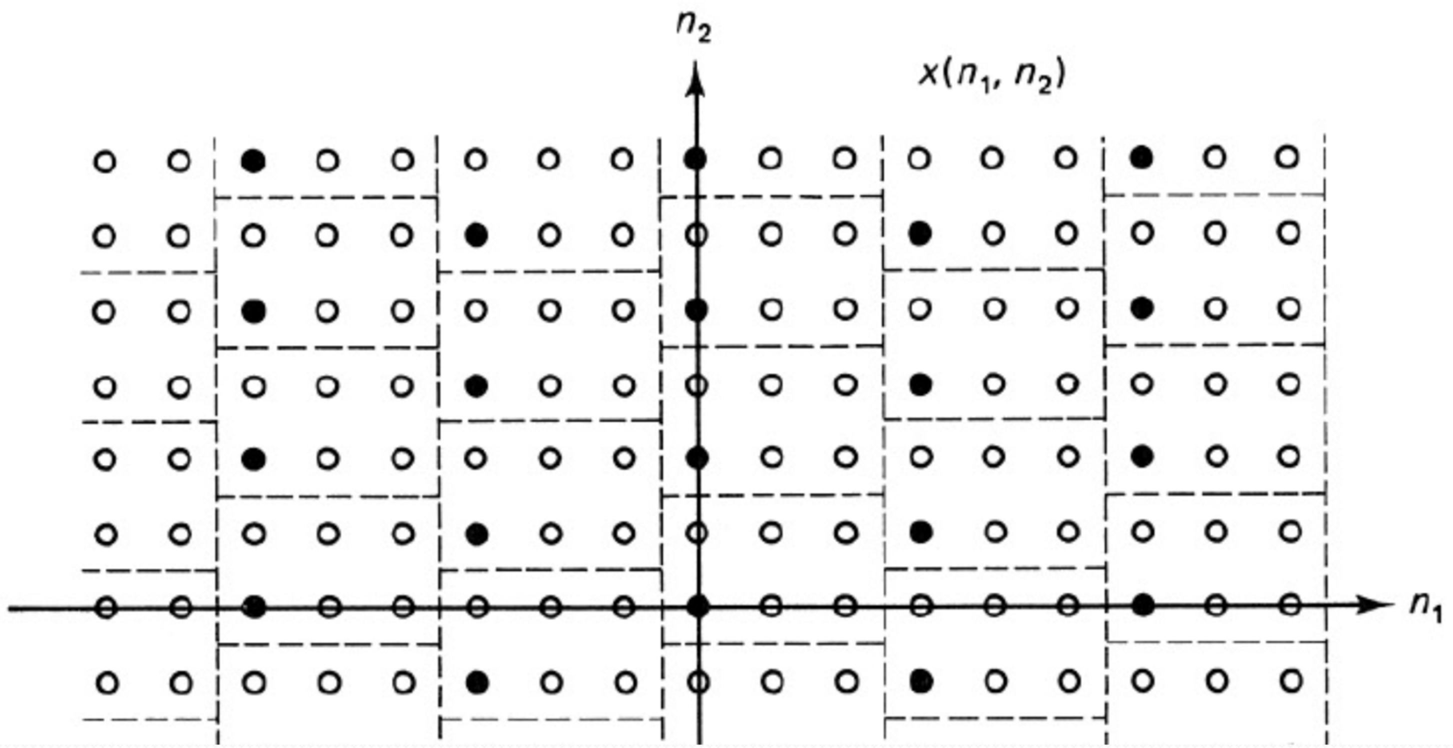
$$\delta(n_1, n_2) = \delta(n_1)\,\delta(n_2)$$

$$u(n_1, n_2) = u(n_1)u(n_2)$$

Periodic sequences

- A sequence $x(n_1, n_2)$ is periodic of period N_1 x N_2 if:

$$x(n_1, n_2) = x(n_1 + N_1, n_2) = x(n_1, n_2 + N_2) \quad \text{for all } (n_1, n_2)$$



LTI Systems

- Linearity

$$T[ax_1(n_1, n_2) + bx_2(n_1, n_2)] = ay_1(n_1, n_2) + by_2(n_1, n_2)$$

- Spatial invariance

$$T[x(n_1 - m_1, n_2 - m_2)] = y(n_1 - m_1, n_2 - m_2)$$

- The impulse response

$$y(n_1,n_2) = T[x(n_1,n_2)] = T\left[\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x(k_1,k_2)\,\delta(n_1-k_1,n_2-k_2)\right]$$

$$= \sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x(k_1,k_2)T[\delta(n_1-k_1,n_2-k_2)].$$

Convolution

- Defined the impulse response

$$h(n_1, n_2) = T[\delta(n_1, n_2)].$$

- The Input/Output relation is given by:

$$y(n_1, n_2) = T[x(n_1, n_2)] = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)h(n_1 - k_1, n_2 - k_2).$$

$$y(n_1, n_2) = x(n_1, n_2) * h(n_1, n_2)$$

$$= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)h(n_1 - k_1, n_2 - k_2).$$

Convolution properties

Commutativity

$$x(n_1, n_2) * y(n_1, n_2) = y(n_1, n_2) * x(n_1, n_2)$$

Associativity

$$(x(n_1, n_2) * y(n_1, n_2)) * z(n_1, n_2) = x(n_1, n_2) * (y(n_1, n_2) * z(n_1, n_2))$$

Distributivity

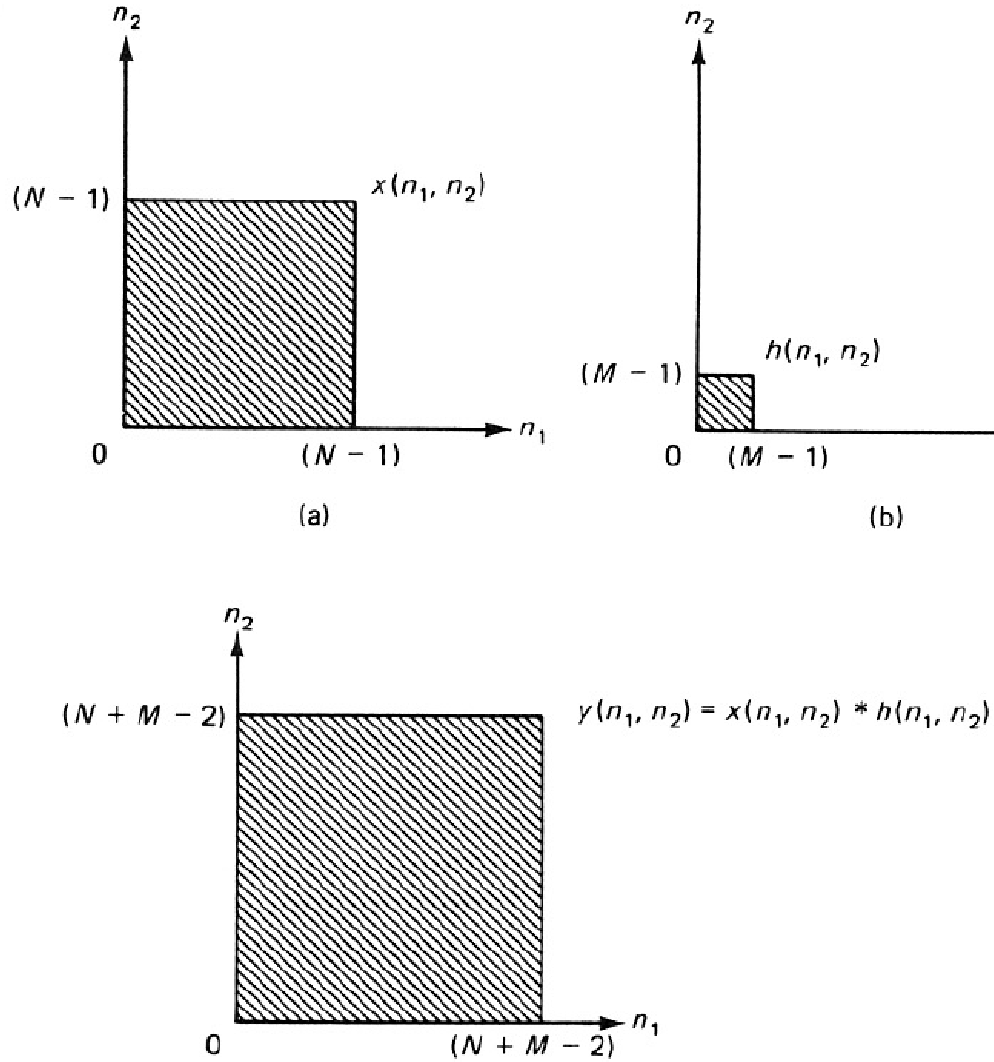
$$x(n_1, n_2) * (y(n_1, n_2) + z(n_1, n_2))$$

$$= (x(n_1, n_2) * y(n_1, n_2)) + (x(n_1, n_2) * z(n_1, n_2))$$

Convolution with Shifted Impulse

$$x(n_1, n_2) * \delta(n_1 - m_1, n_2 - m_2) = x(n_1 - m_1, n_2 - m_2)$$

Convolution examples



The 2D Fourier Transform

- The analysis and synthesis formulas for the 2D continuous Fourier transform are as follows:
- Analysis

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dx\,dy$$

- Synthesis

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}du\,dv$$

Separability of 2D Fourier Transform

- The 2D analysis formula can be written as a 1D analysis in the *x* direction followed by a *1D* analysis in the *y* direction:

$$F(u,v)=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}f(x,y)e^{-j2\pi ux}dx\right]e^{-j2\pi vy}dy$$

- The 2D synthesis formula can be written as a 1D synthesis in the *x* direction followed by a 1D synthesis in *y* direction:

$$f(x,y)=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}F(u,v)e^{j2\pi ux}du\right]e^{j2\pi vy}dv$$

Separability Theorem

$$f(x,y) = f(x)g(y) \xrightarrow{\mathcal{F}} F(u,v) = F(u)G(v)$$

Proof:

$$F(u,v)$$

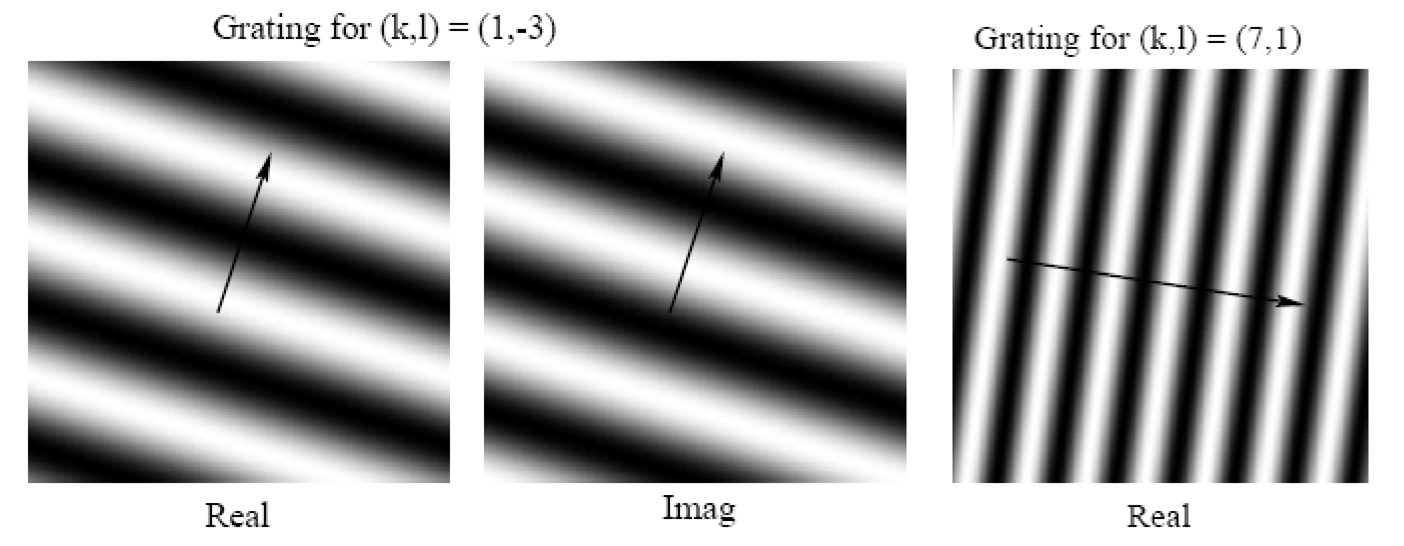
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dx\,dy$$

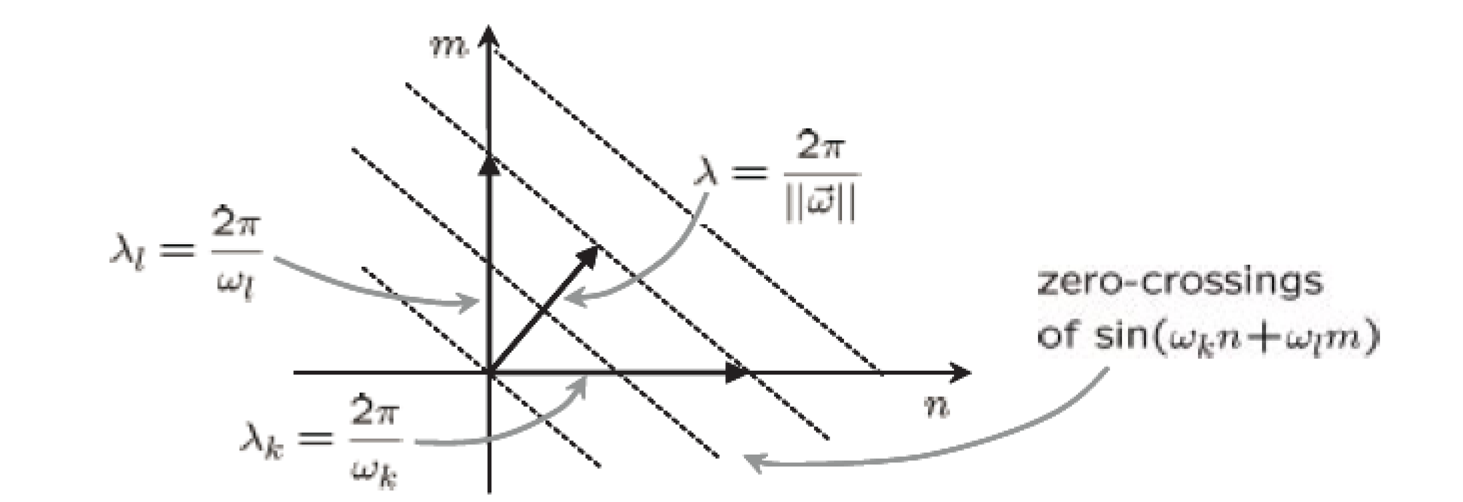
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(y)e^{-j2\pi ux}e^{-j2\pi vy}\,dx\,dy$$

$$= \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}\,dx \int_{-\infty}^{\infty} g(y)e^{-j2\pi vy}dy$$

$$= F(u)\,G(v)$$

2D Fourier Basis Functions





The 2D Discrete Fourier Transform for periodic signals

- The analysis and synthesis formulas for the 2D discrete Fourier transform are as follows:
- $$\hat{F}(k,\ell) = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} F(m,n) e^{-j2\pi\left(k\frac{m}{M}+\ell\frac{n}{N}\right)}$$

$$F(m,n) = \frac{1}{\sqrt{MN}} \sum_{k=0}^{M-1} \sum_{\ell=0}^{N-1} \hat{F}(k,\ell) e^{j2\pi\left(k\frac{m}{M}+\ell\frac{n}{N}\right)}$$

Separability of 2D DFT

$$\hat{F}(k,\ell) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left[\frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} F(m,n) e^{-j2\pi\left(k\frac{m}{M}\right)} \right] e^{-j2\pi\left(\ell\frac{n}{N}\right)}$$

- The 2D forward DFT can be written in matrix notation: $\hat{\mathbf{F}} = (\mathbf{W}^{*}\mathbf{F})\,\mathbf{W}^{*}$
- Where $W_{rc}^{*} = \frac{1}{\sqrt{C}} e^{-j2\pi r\frac{c}{C}}$

Separability of 2D DFT

- And

$$F(m,n) = \frac{1}{\sqrt{N}} \sum_{\ell=0}^{N-1} \left[\frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \hat{F}(k,\ell) e^{j2\pi\left(k\frac{m}{M}\right)} \right] e^{j2\pi\left(\ell\frac{n}{N}\right)}.$$

- The 2D inverse DFT can be written in matrix notation:

$$\mathbf{F} = \left(\mathbf{W}\hat{\mathbf{F}}\right)\mathbf{W}$$

- where

$$W_{rc} = \frac{1}{\sqrt{C}} e^{j2\pi r \frac{c}{C}}$$

The 2D Discrete Time Fourier Transform

Discrete-Space Fourier Transform Pair

$$X(\omega_1, \omega_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$$x(n_1, n_2) = \frac{1}{(2\pi)^2} \int_{\omega_1=-\pi}^{\pi} \int_{\omega_2=-\pi}^{\pi} X(\omega_1, \omega_2) e^{j\omega_1 n_1} e^{j\omega_2 n_2} \, d\omega_1 \, d\omega_2$$

La trasformata di un segnale può essere espresso come componente reale e immaginaria o come ampiezza e fase.

$$X(\omega_1, \omega_2) = |X(\omega_1, \omega_2)| e^{j\theta_x(\omega_1, \omega_2)} = X_R(\omega_1, \omega_2) + jX_I(\omega_1, \omega_2).$$

Fourier Transform properties

$$x(n_1, n_2) \longleftrightarrow X(\omega_1, \omega_2)$$
$$y(n_1, n_2) \longleftrightarrow Y(\omega_1, \omega_2)$$

Property 1. *Linearity*

$$ax(n_1, n_2) + by(n_1, n_2) \longleftrightarrow aX(\omega_1, \omega_2) + bY(\omega_1, \omega_2)$$

Property 2. *Convolution*

$$x(n_1, n_2) * y(n_1, n_2) \longleftrightarrow X(\omega_1, \omega_2)Y(\omega_1, \omega_2)$$

Property 3. *Multiplication*

$$x(n_1, n_2)y(n_1, n_2) \longleftrightarrow X(\omega_1, \omega_2) \circledast Y(\omega_1, \omega_2)$$

$$= \frac{1}{(2\pi)^2} \int_{\theta_1=-\pi}^{\pi} \int_{\theta_2=-\pi}^{\pi} X(\theta_1, \theta_2)Y(\omega_1 - \theta_1, \omega_2 - \theta_2)\, d\theta_1\, d\theta_2$$

Property 4. *Separable Sequence*

$$x(n_1, n_2) = x_1(n_1)x_2(n_2) \longleftrightarrow X(\omega_1, \omega_2) = X_1(\omega_1)X_2(\omega_2)$$

Property 5. *Shift of a Sequence and a Fourier Transform*

(a) $x(n_1 - m_1, n_2 - m_2) \longleftrightarrow X(\omega_1, \omega_2)e^{-j\omega_1 m_1}e^{-j\omega_2 m_2}$

(b) $e^{j\nu_1 n_1}e^{j\nu_2 n_2}x(n_1, n_2) \longleftrightarrow X(\omega_1 - \nu_1, \omega_2 - \nu_2)$

Fourier Transform properties

Property 6. Differentiation

(a) $-jn_1x(n_1, n_2) \longleftrightarrow \dfrac{\partial X(\omega_1, \omega_2)}{\partial \omega_1}$

(b) $-jn_2x(n_1, n_2) \longleftrightarrow \dfrac{\partial X(\omega_1, \omega_2)}{\partial \omega_2}$

Property 7. Initial Value and DC Value Theorem

(a) $x(0, 0) = \dfrac{1}{(2\pi)^2} \displaystyle\int_{\omega_1=-\pi}^{\pi} \int_{\omega_2=-\pi}^{\pi} X(\omega_1, \omega_2)\, d\omega_1\, d\omega_2$

(b) $X(0, 0) = \displaystyle\sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x(n_1, n_2)$

Property 8. Parseval's Theorem

(a) $$\sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x(n_1, n_2)y^*(n_1, n_2) = \frac{1}{(2\pi)^2} \int_{\omega_1=-\pi}^{\pi} \int_{\omega_2=-\pi}^{\pi} X(\omega_1, \omega_2)Y^*(\omega_1, \omega_2)\, d\omega_1\, d\omega_2$$

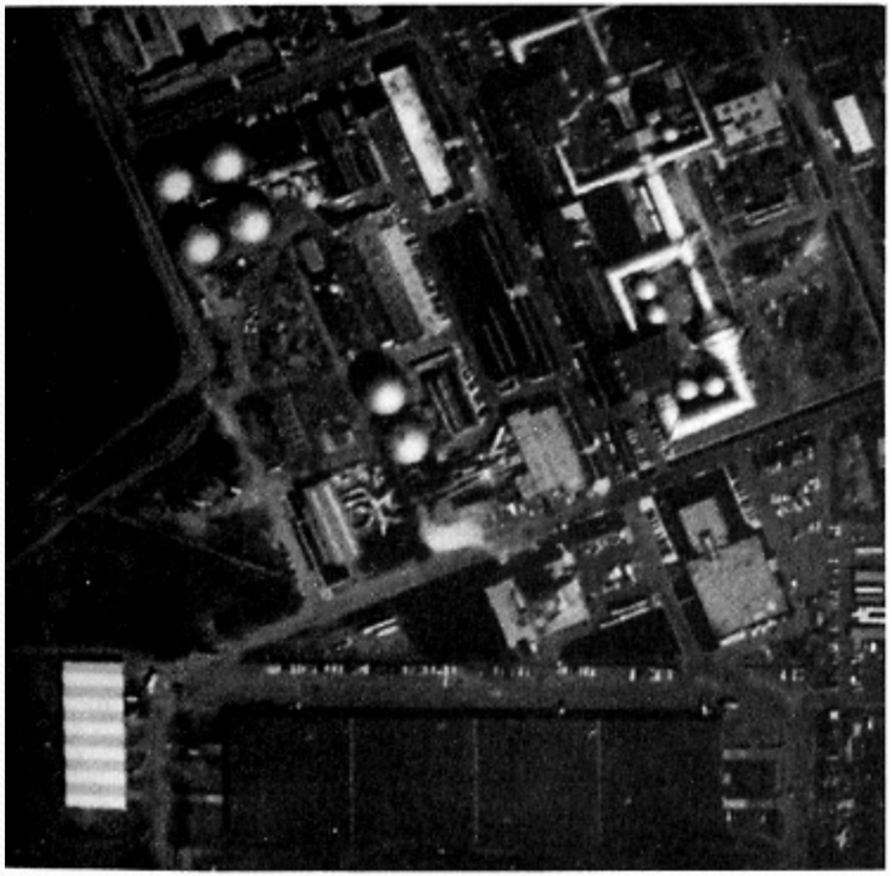
(b) $$\sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} |x(n_1, n_2)|^2 = \frac{1}{(2\pi)^2} \int_{\omega_1=-\pi}^{\pi} \int_{\omega_2=-\pi}^{\pi} |X(\omega_1, \omega_2)|^2\, d\omega_1\, d\omega_2$$

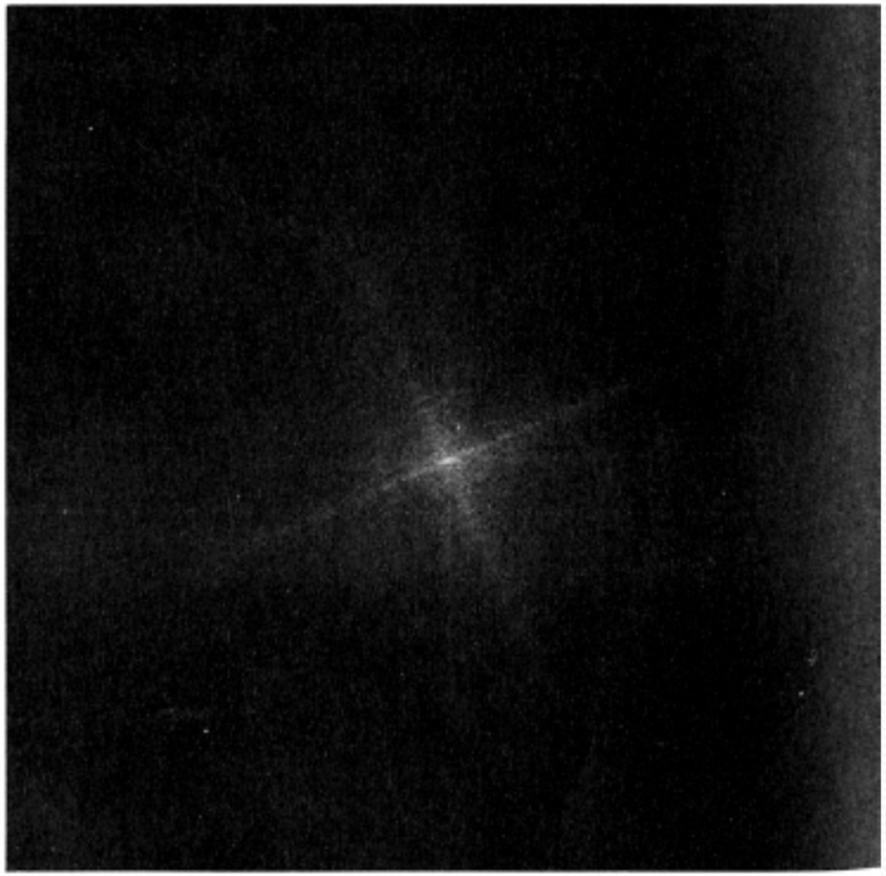
Fourier Transform properties

Property 9. Symmetry Properties

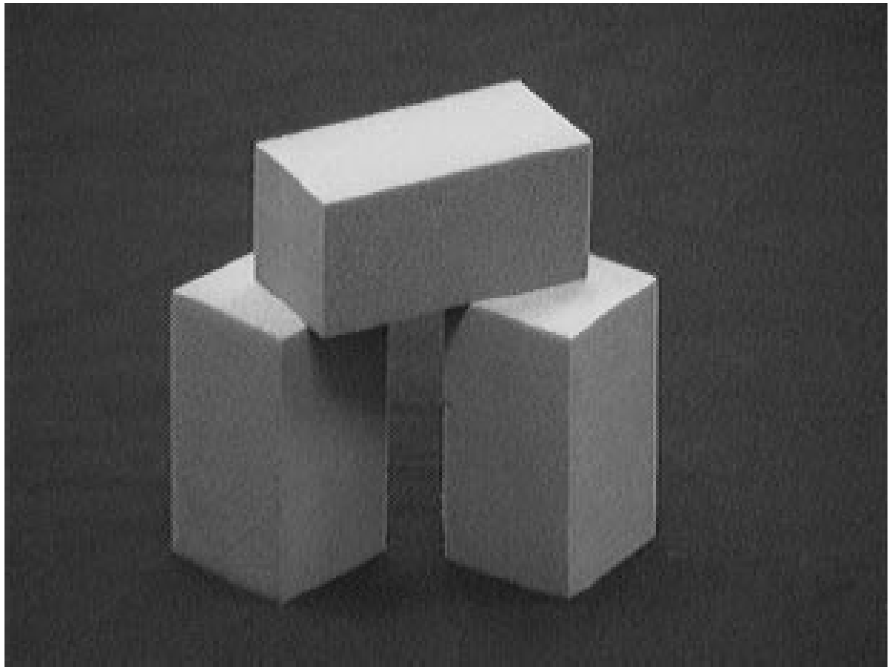
(a) $x(-n_1, n_2) \longleftrightarrow X(-\omega_1, \omega_2)$
(b) $x(n_1, -n_2) \longleftrightarrow X(\omega_1, -\omega_2)$
(c) $x(-n_1, -n_2) \longleftrightarrow X(-\omega_1, -\omega_2)$
(d) $x^*(n_1, n_2) \longleftrightarrow X^*(-\omega_1, -\omega_2)$
(e) $x(n_1, n_2)$: real $\longleftrightarrow X(\omega_1, \omega_2) = X^*(-\omega_1, -\omega_2)$
$X_R(\omega_1, \omega_2)$, $|X(\omega_1, \omega_2)|$: even (symmetric with respect to the origin)
$X_I(\omega_1, \omega_2)$, $\theta_x(\omega_1, \omega_2)$: odd (antisymmetric with respect to the origin)
(f) $x(n_1, n_2)$: real and even $\longleftrightarrow X(\omega_1, \omega_2)$: real and even
(g) $x(n_1, n_2)$: real and odd $\longleftrightarrow X(\omega_1, \omega_2)$: pure imaginary and odd

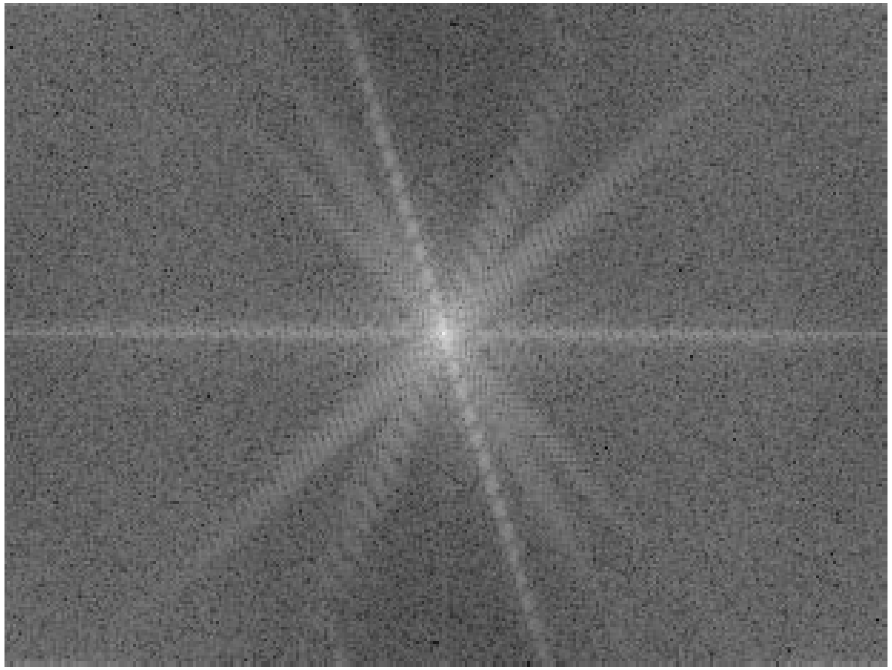
Transform examples



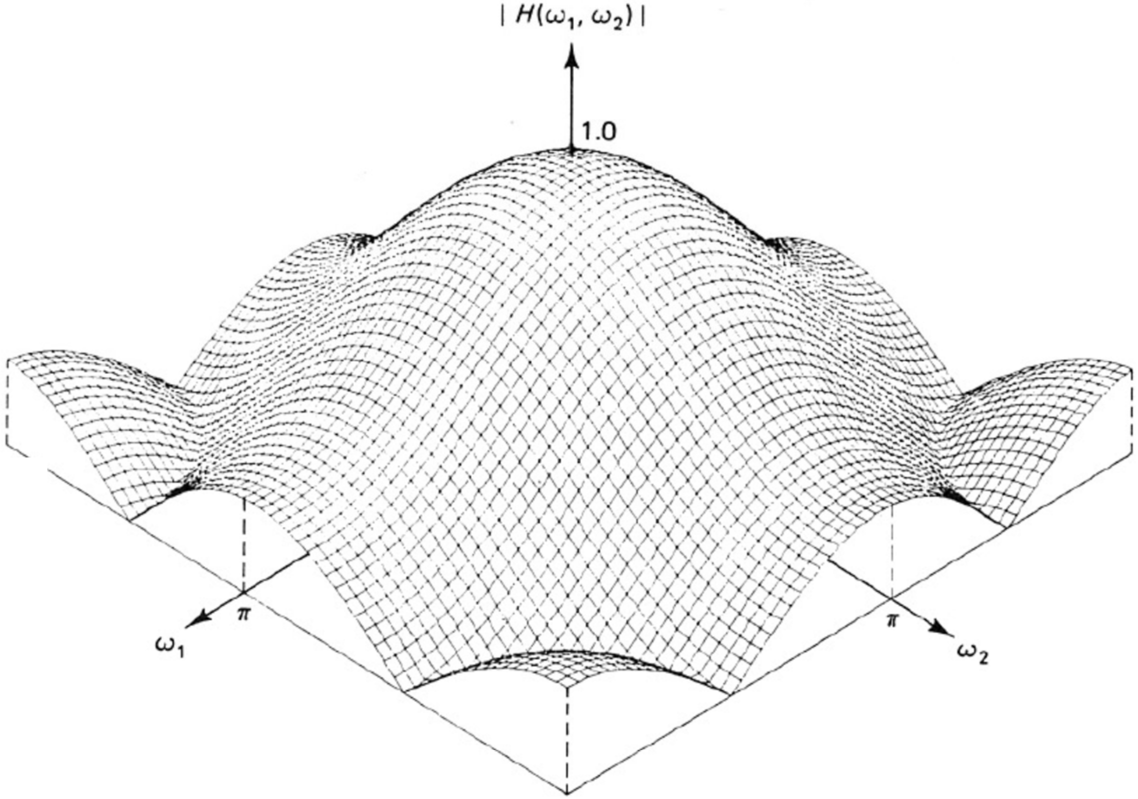


2D DFT Example

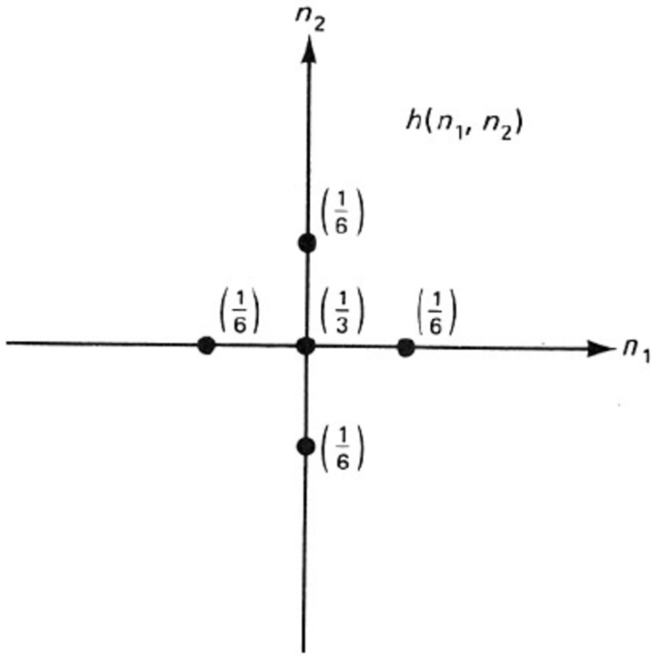




DTFT exam

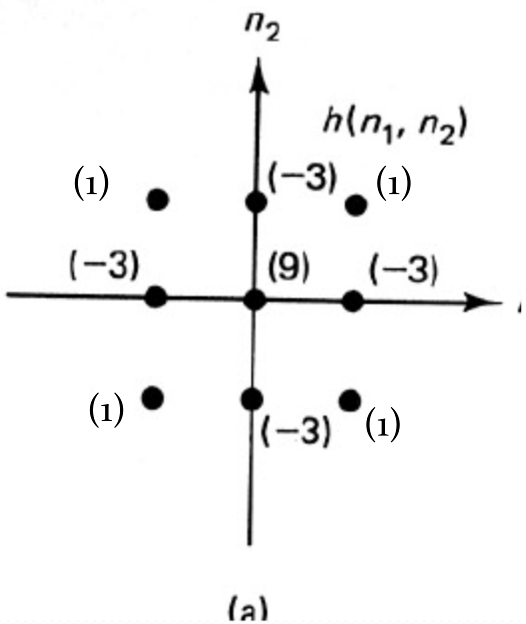


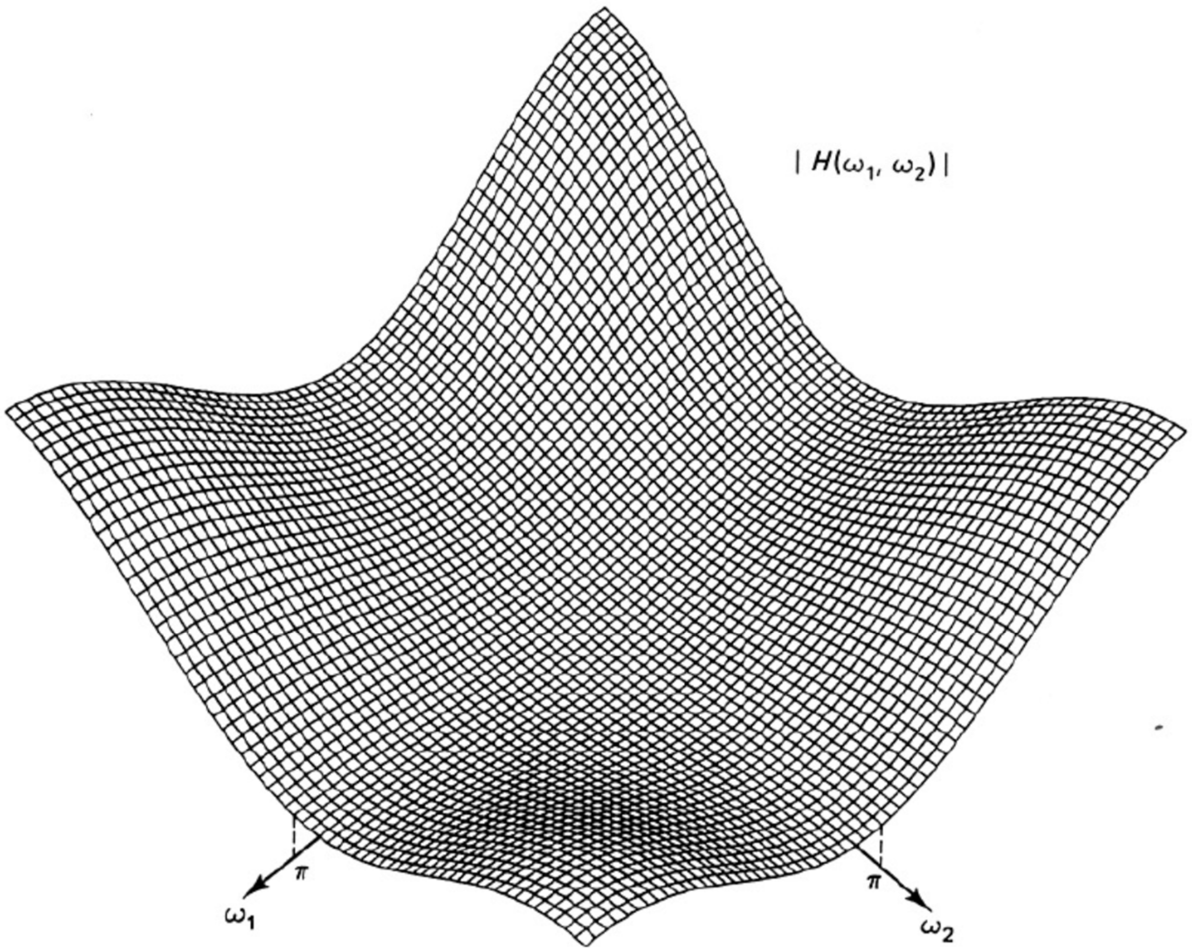
$|H(\omega_1, \omega_2)|$
1.0
π
ω_1
π
ω_2



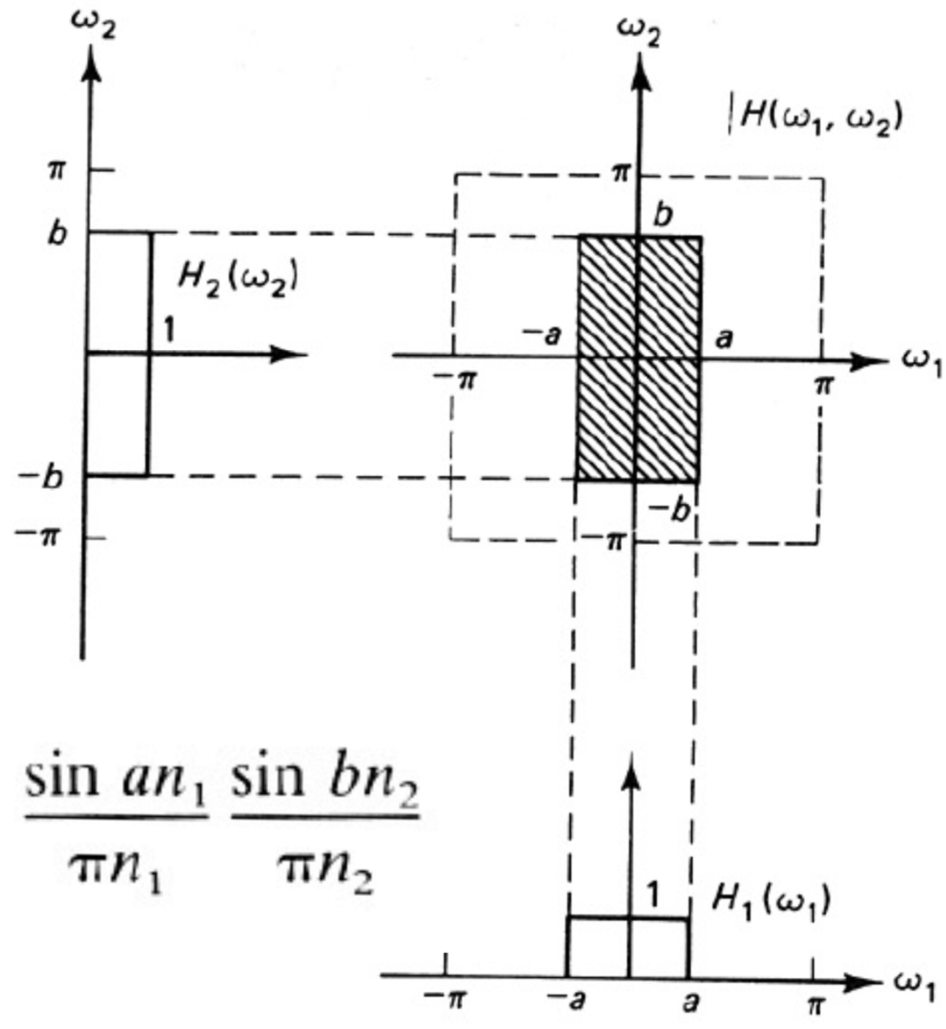
n_2
$h(n_1, n_2)$
$\left(\frac{1}{6}\right)$
$\left(\frac{1}{6}\right)$
$\left(\frac{1}{3}\right)$
$\left(\frac{1}{6}\right)$
n_1
$\left(\frac{1}{6}\right)$

Hi-Pass Filter example





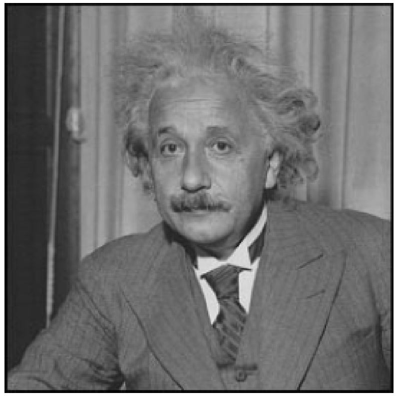
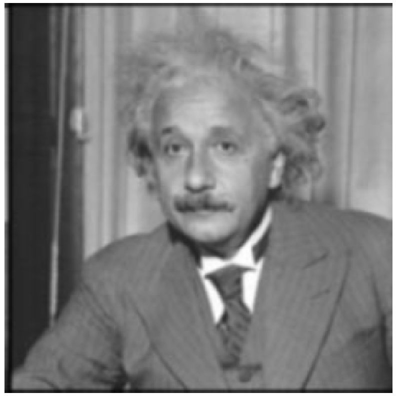
Separable Low-pass Filter example

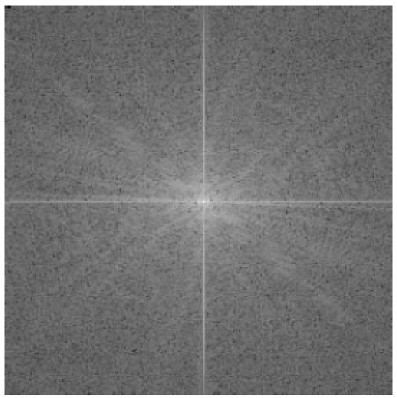


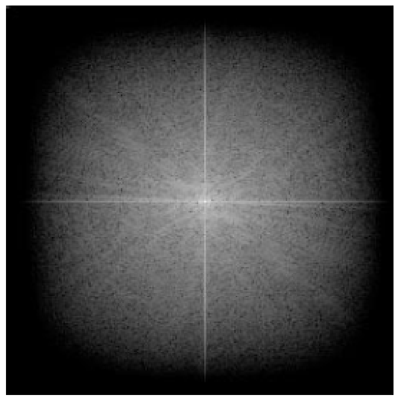
$$h(n_1, n_2) = h_1(n_1)h_2(n_2) = \frac{\sin an_1}{\pi n_1}\frac{\sin bn_2}{\pi n_2}$$

Image of Albert and a low-pass (blurred) version of it

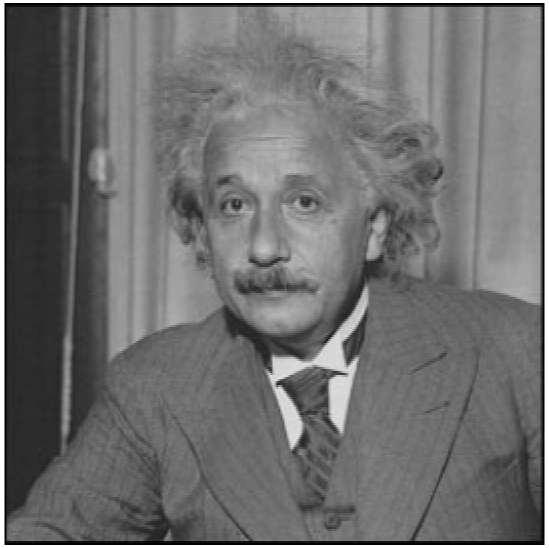
- $h(n) = \frac{1}{16}(1,4,6,4,1)$ (1D impulse response in both dimensions)

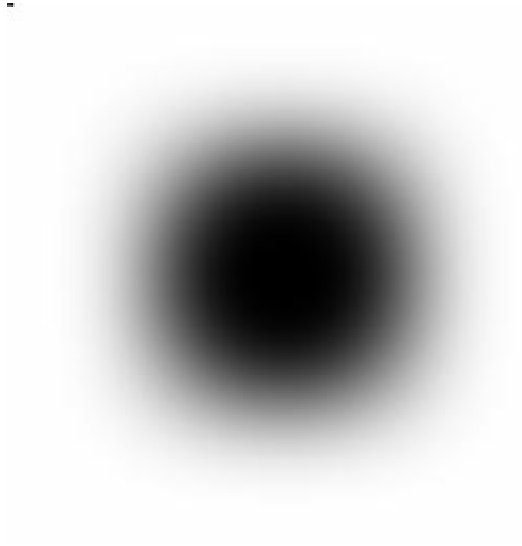







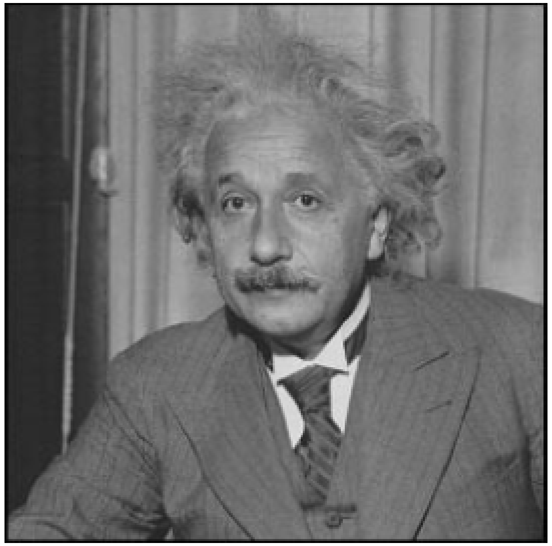
Hi-Pass Filter

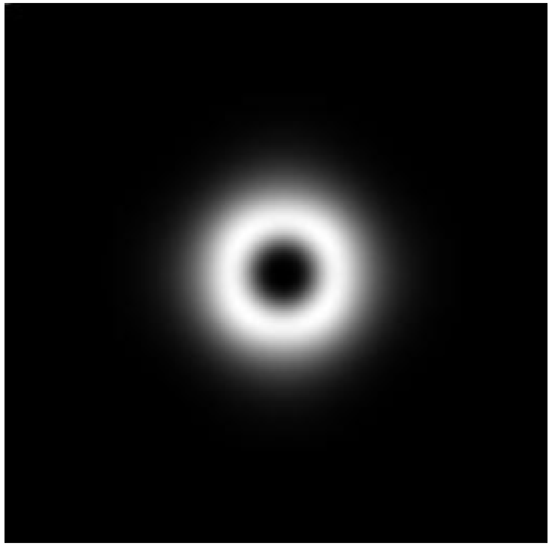




- a high-pass filtered version of Albert, and the amplitude spectrum of the filter. This impulse response is defined by $\delta(n)-h(n,m)$ where $h(n,m)$ is the separable blurring kernel used in the previous figure

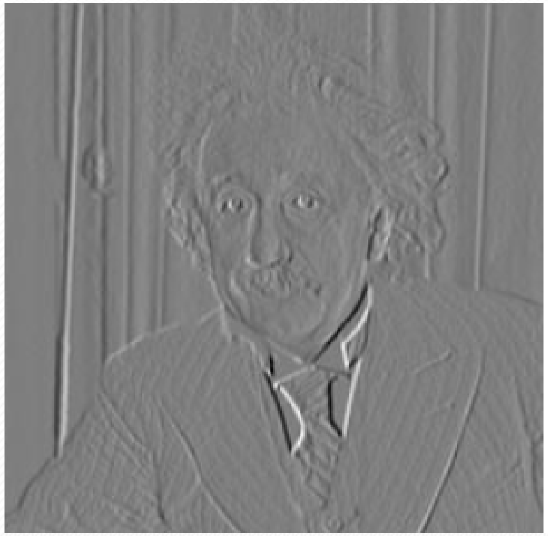
Band Pass Filter

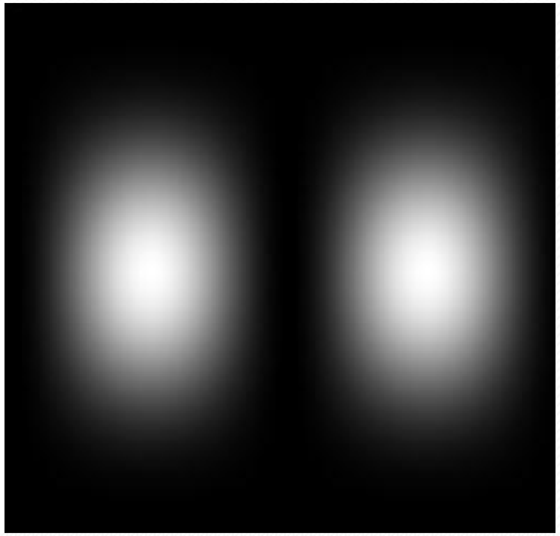


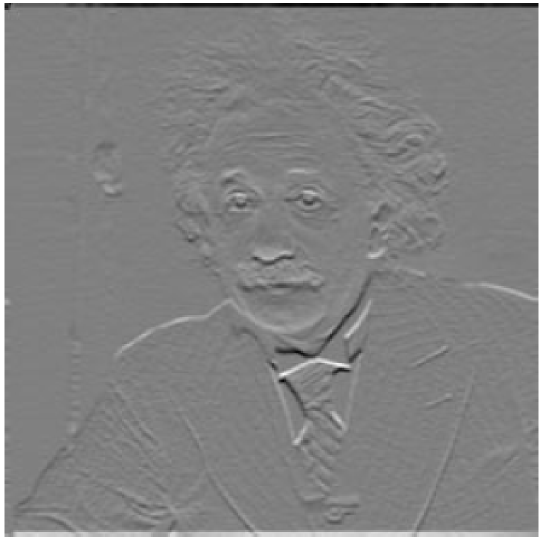


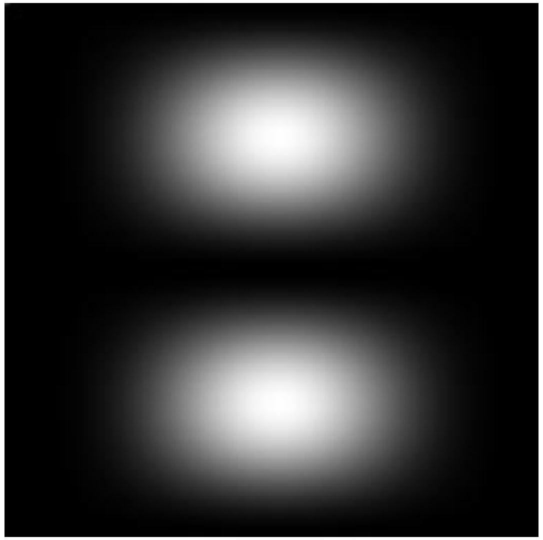
- a band-pass filtered version of Albert, and the amplitude spectrum of the filter.
- This impulse response is defined by the difference of two low-pass filters.

Directional filters

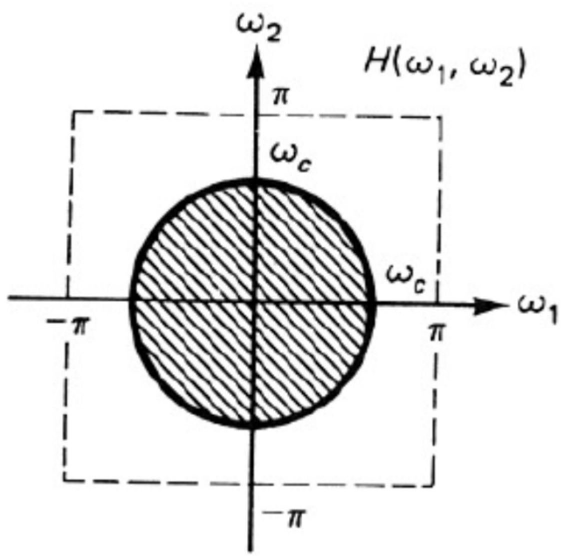


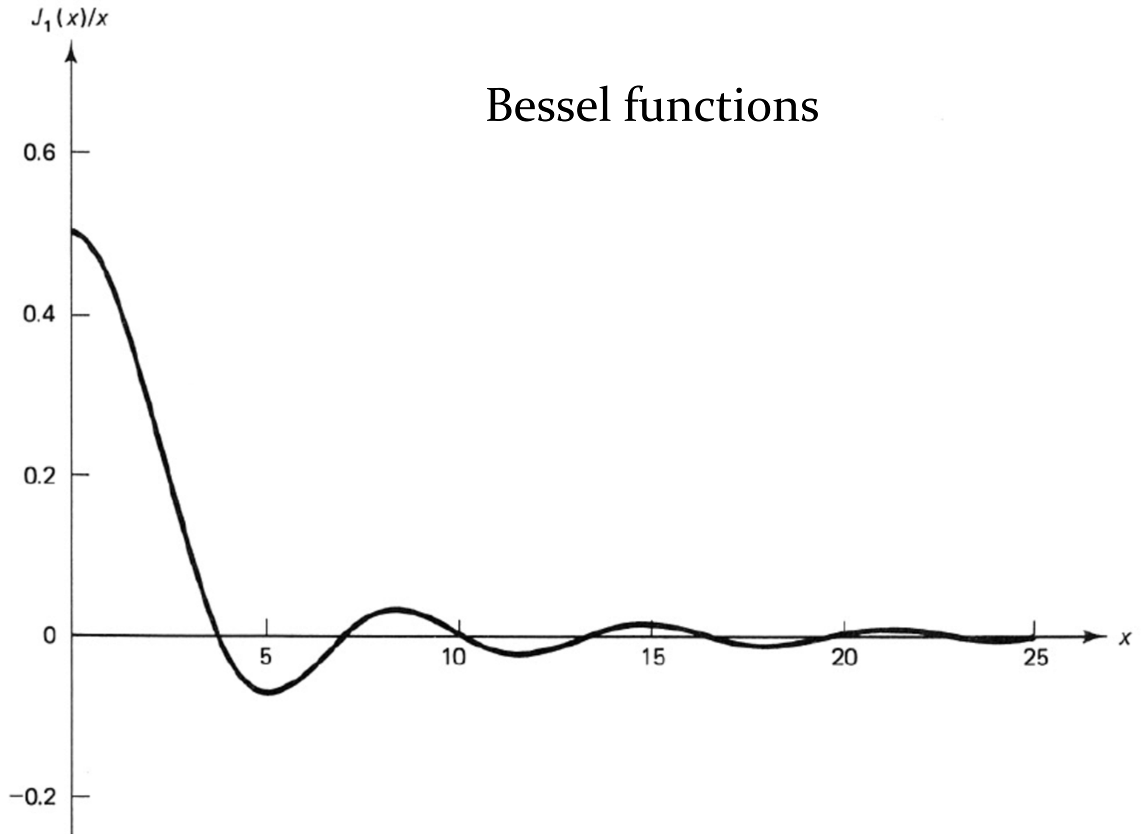






Circular filter example





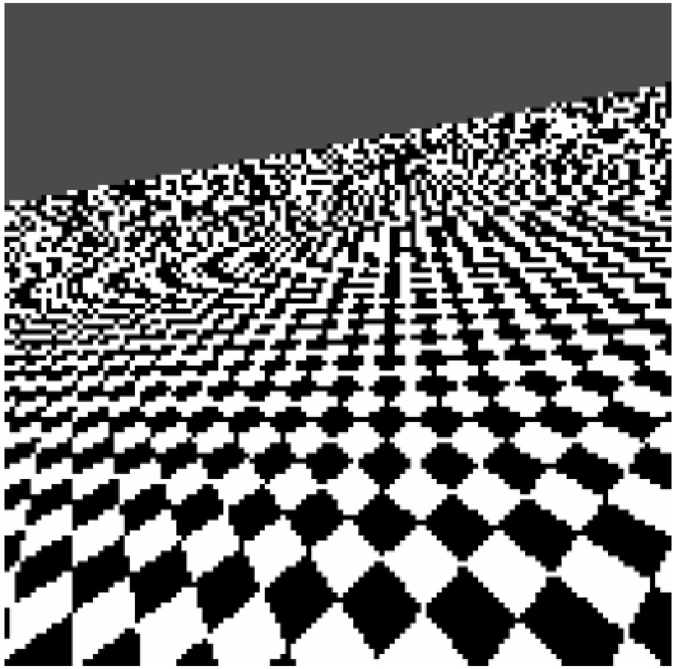
$$h(n_1, n_2) = \frac{\omega_C}{2\pi\sqrt{n_1^2 + n_2^2}} J_1\left(\omega_C \sqrt{n_1^2 + n_2^2}\right)$$

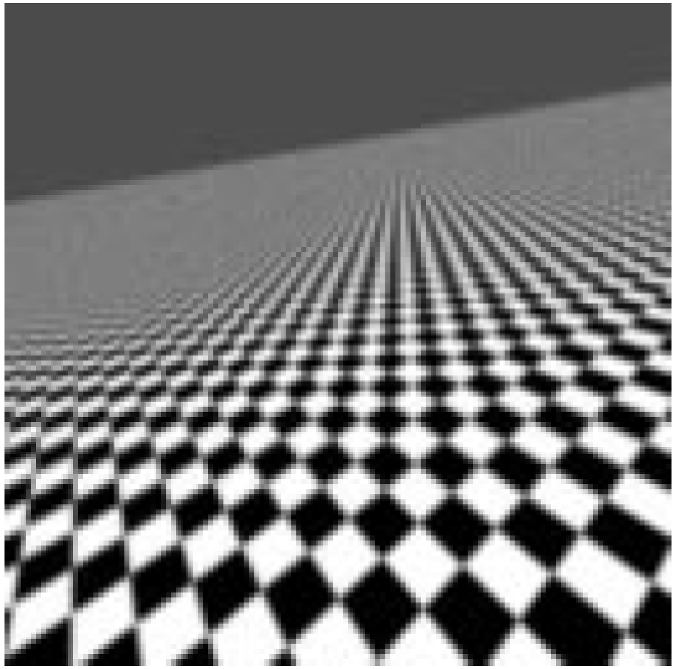
Nyquist Sampling Theorem and Aliasing

- Consider a perspective image of an infinite checkerboard.
- The signal is dominated by high frequencies in the image near the horizon.
- Properly designed cameras blur the signal before sampling using:
  - The point spread function due to diffraction
  - Imperfect focus
  - Averaging the signal over each CCD element.

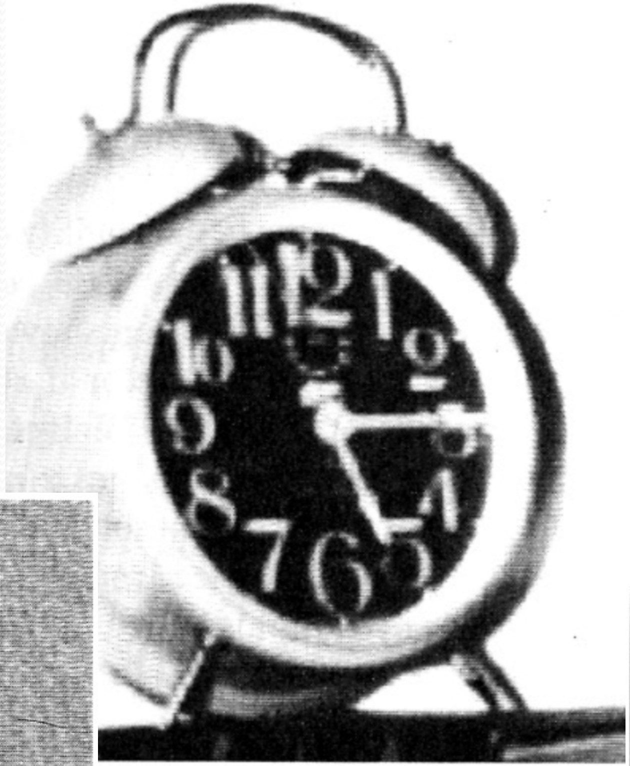
Nyquist Sampling Theorem and Aliasing

- These operations attenuate high frequency components in the signal.
- Without this (physical) preprocessing, the sampled image can be severely aliased (corrupted):





Reconstruction using just phase or intensity



Only phase

Only amplitude

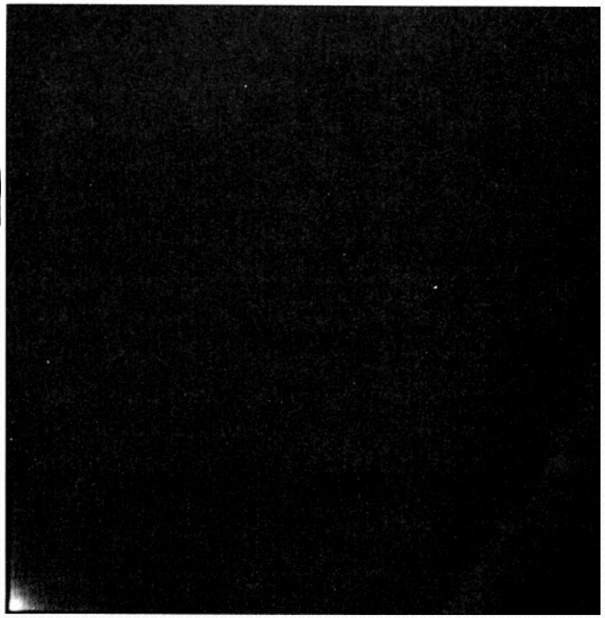
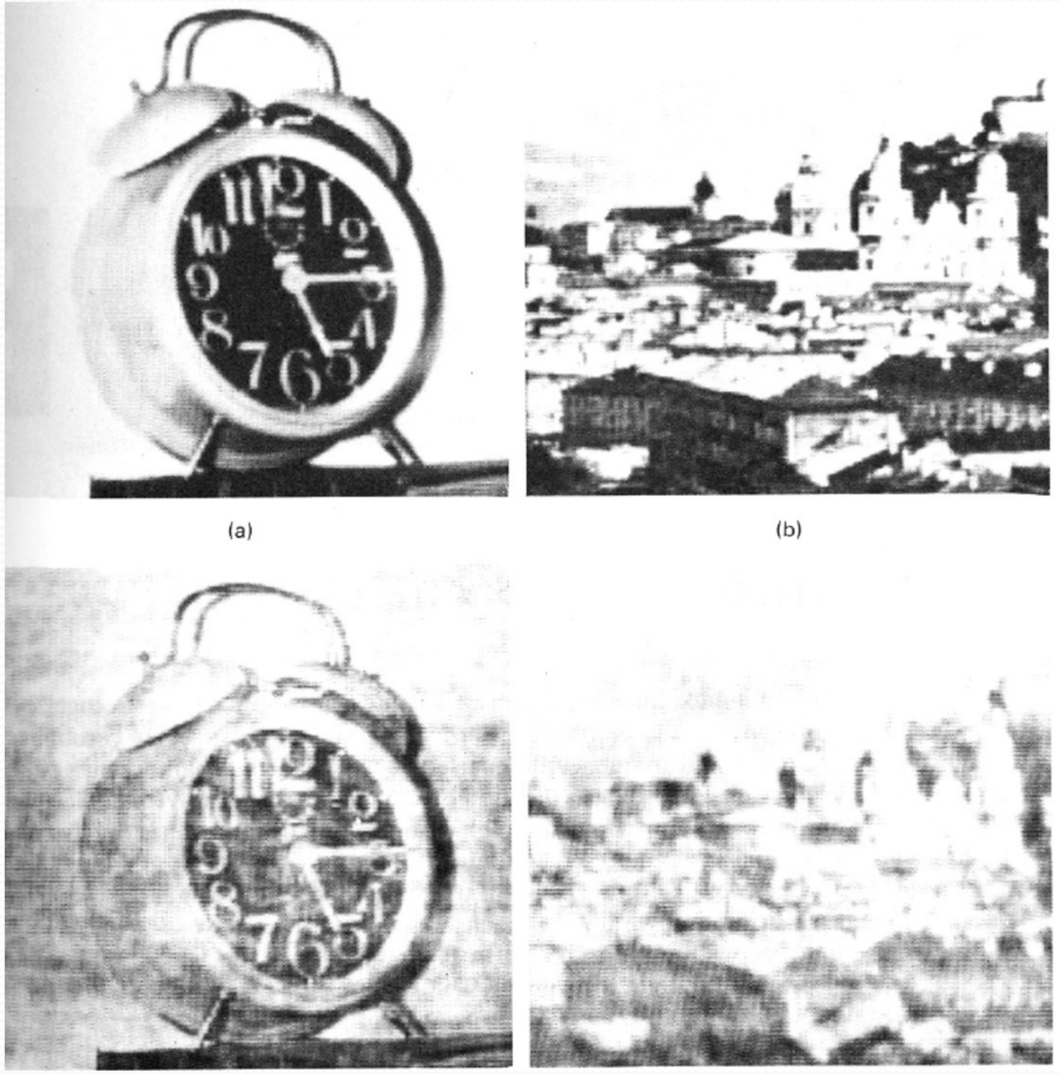


Image superposition combining phases and intesities



Filtering examples

Original
Cameraman

Cameraman blurred by convolution

Filter impulse response

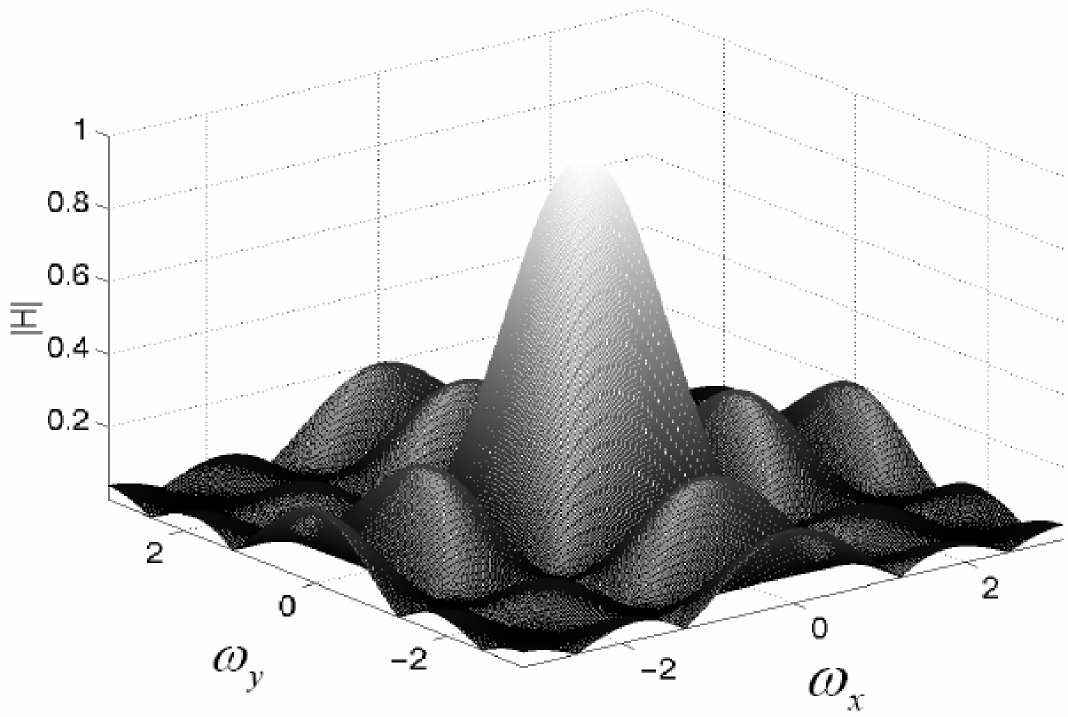
$$\frac{1}{25}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & [1] & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Fourier interpretation

$$H\left(e^{j\omega_x}, e^{j\omega_y}\right) = \sum_{m-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h[m,n] e^{-j\omega_x m - j\omega_y n}$$

$$= \frac{1}{25} \sum_{m=-2}^{2} \sum_{n=-2}^{2} e^{-j\omega_x m - j\omega_y n} = \frac{1}{25} \sum_{m=-2}^{2} e^{-j\omega_x m} \sum_{n=-2}^{2} e^{-j\omega_y n}$$

$$= \frac{1}{25}\left(1 + 2\cos\omega_x + 2\cos(2\omega_x)\right)\left(1 + 2\cos\omega_y + 2\cos(2\omega_y)\right)$$



Filtering Examples

Original
Cameraman

Cameraman blurred horizontally

Filter impulse response

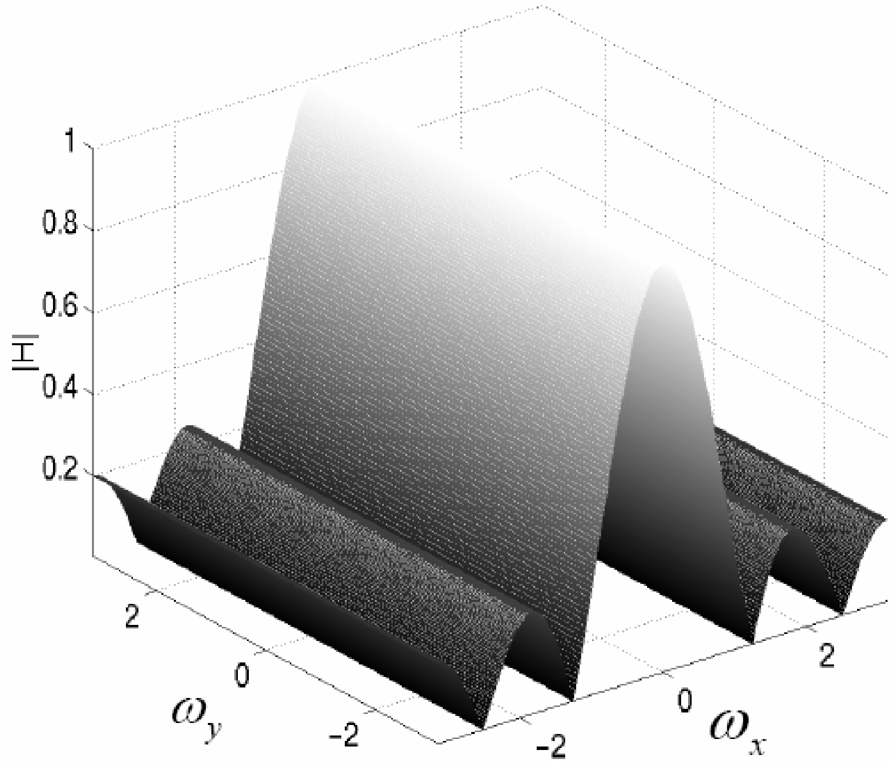
$$\frac{1}{5}\begin{pmatrix}1 & 1 & [1] & 1 & 1\end{pmatrix}$$

Fourier interpretation

Cameraman blurred horizontally

Filter impulse response

$$\frac{1}{5}\begin{pmatrix}1 & 1 & [1] & 1 & 1\end{pmatrix}$$



Filtering examples

Original
Cameraman

Cameraman blurred vertically
Filter impulse response

$$\frac{1}{5}\begin{pmatrix} 1 \\ 1 \\ [1] \\ 1 \\ 1 \end{pmatrix}$$

Filtering examples

Original
Cameraman

Cameraman sharpened

Filter impulse response

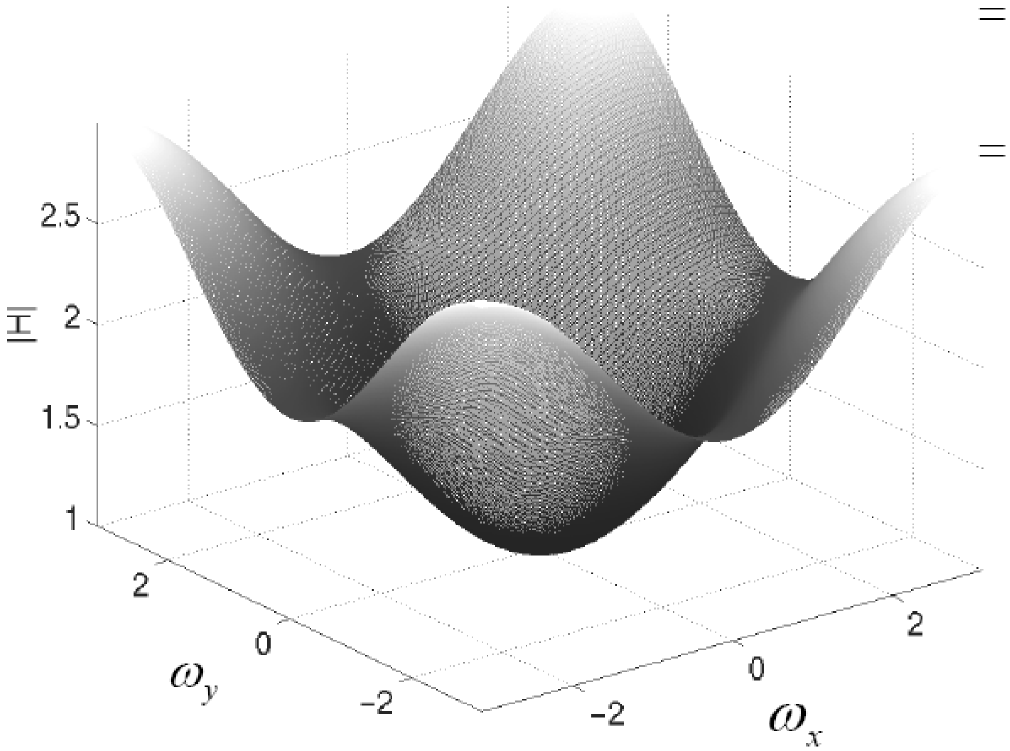
$$\frac{1}{4}\begin{pmatrix} 0 & -1 & 0 \\ -1 & [8] & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Fourier interpretation

$$H\left(e^{j\omega_x}, e^{j\omega_y}\right) = \sum_{m-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h[m,n] e^{-j\omega_x m - j\omega_y n}$$

$$= \frac{1}{4}\left(8 - e^{-j\omega_x} - e^{j\omega_x} - e^{-j\omega_y} - e^{j\omega_y}\right)$$

$$= 2 - \frac{1}{2}\cos\omega_x - \frac{1}{2}\cos\omega_y$$



Filtering examples

Original
Cameraman

Cameraman sharpened

Filter impulse response

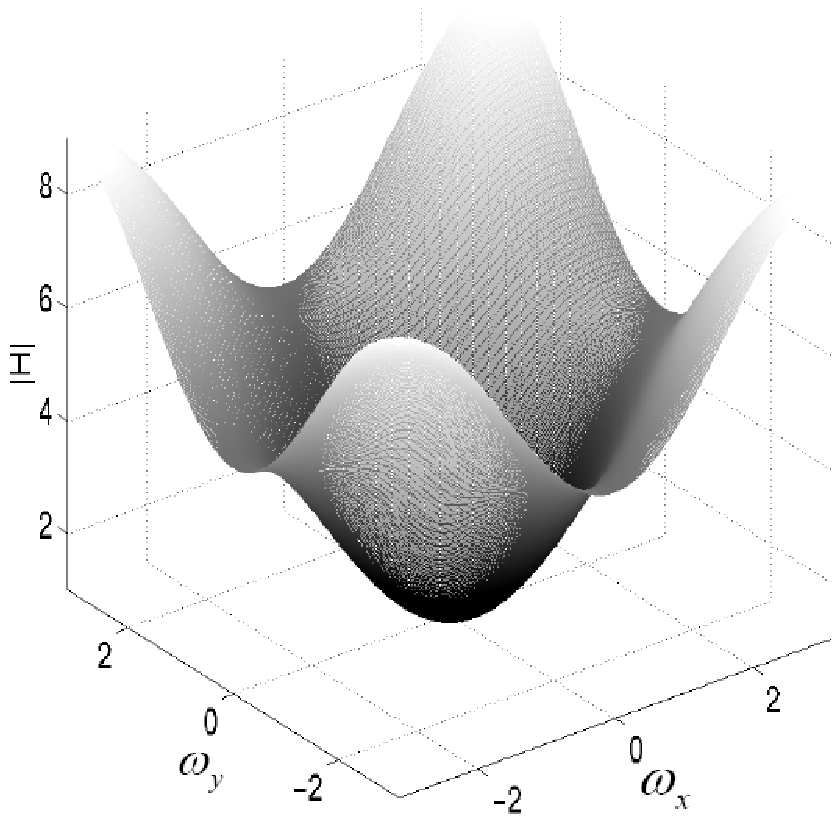
$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & [5] & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Fourier interpretation

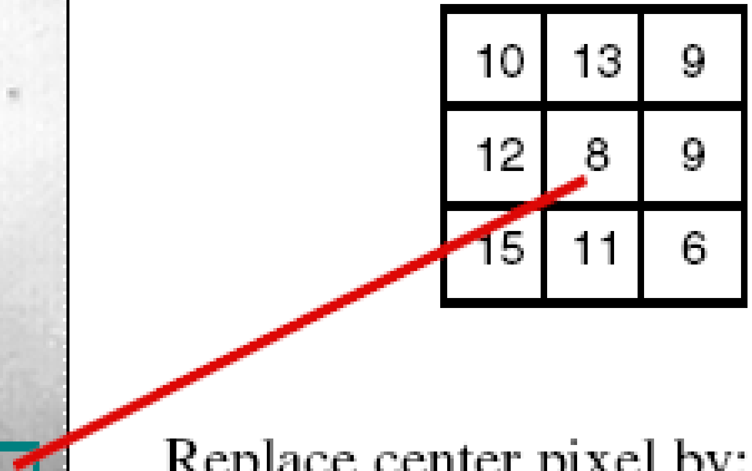
Cameraman sharpened

Filter impulse response

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & [5] & -1 \\ 0 & -1 & 0 \end{pmatrix}$$



Linear and non linear operations



Replace center pixel by:

Median Filter: (6, 8, 9, 9,**10**, 11, 12, 13, 15) = 10

Minimum = 6; Maximum: 15

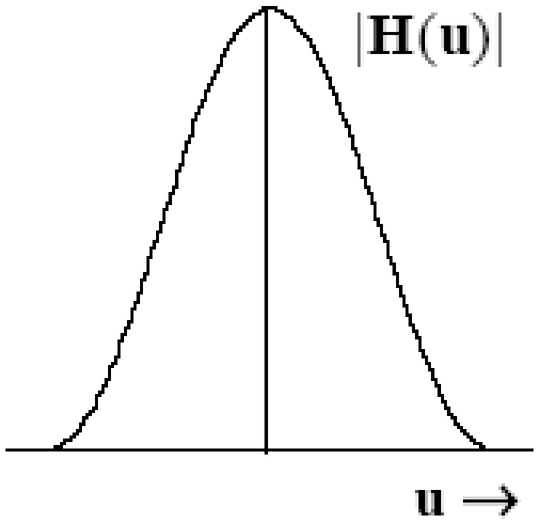
Average of nearest neighbours:

(10+13+9+12+8+9+15+11+6)/9 = 10.33 → **10**

Low pass gaussian filter

$$\frac{1}{16}\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} * \frac{1}{4}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\frac{1}{441}\begin{pmatrix} 4 & 10 & 14 & 10 & 4 \\ 10 & 25 & 35 & 25 & 10 \\ 14 & 35 & 49 & 35 & 14 \\ 10 & 25 & 35 & 25 & 10 \\ 4 & 10 & 14 & 10 & 4 \end{pmatrix}$$



Median filtering