Multimedia Signal Processing 1st Module and Fundamentals of Multimedia Signal Processing

date: February 14th, 2024

Ex.1 (Pt.12)

A discrete filter $h(n)$ is based on the following difference equation:

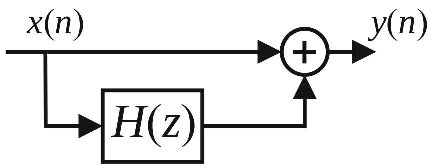
$$y(n)+\frac{1}{4}y(n-2)=\frac{1}{4}x(n)+x(n-2)$$

[3 pts] Draw the pole-zero plot and write the transfer function $H(z)$.

[2 pts] What kind of filter is it?

[3 pts] Draw an approximate amplitude and phase response of the filter.

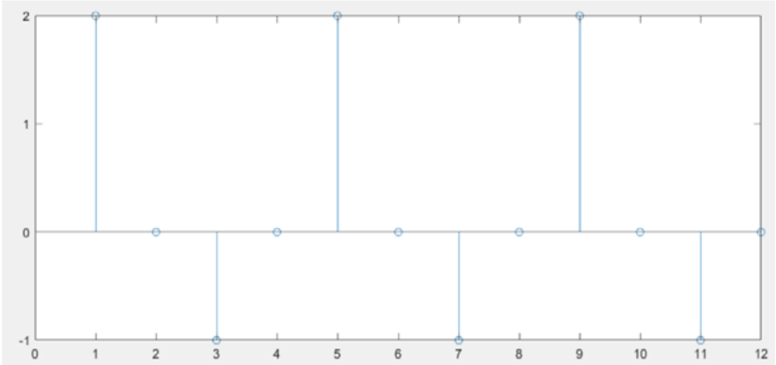
[4 pts] The filter is adopted in the following circuit:



The input signal is: $x(n)=1+\cos\left(\frac{\pi}{2}n\right)+\cos(\pi n)$,

What will be the corresponding output?

Ex.2 (Pt.10)



We want to remove the component at frequency $f=\frac{1}{4}\rightarrow\omega=\frac{\pi}{2}$ from the periodic signal $x(n)$ represented on the right just working with the DFT.

Propose a filter $H(k)$ in the frequency domain and apply it to the signal (provide also the W matrix).

Provide also the output in the time domain $y(n)$ and a period of the filter in the time domain $h(n)$

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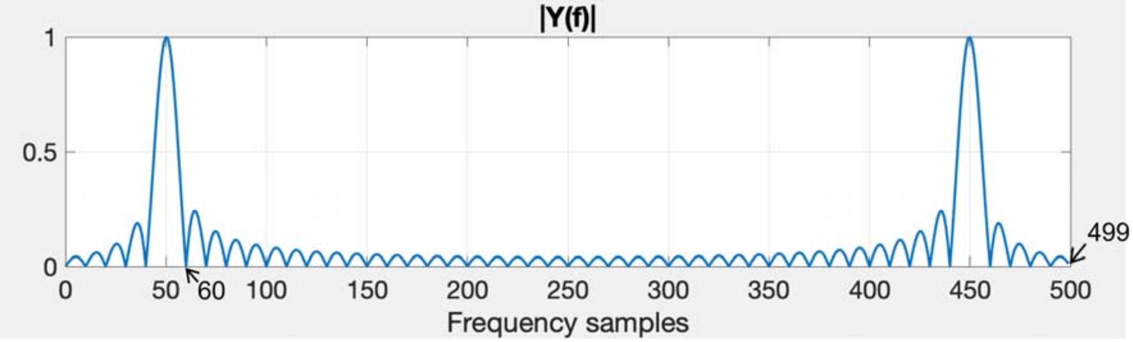
Ex.3 (Pt.11) To be solved writing the MATLAB code on the sheet.

1) [3 pt] In a system working at Fs = 2 KHz, you have the signal y(n) (500 samples) with the following DFT magnitude:



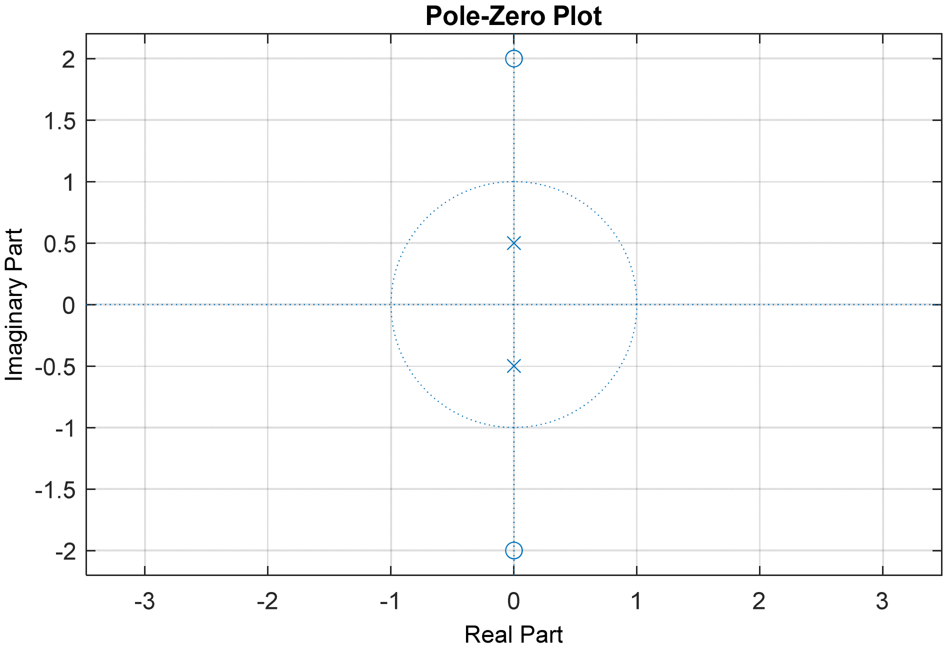
The signal y(n) is the result of zero-padding applied to a cosinusoidal signal x(n).

- Define the signal y(n) starting from x(n) followed by zero-padding. Hint: which is the original length of x(n)? It can be derived from the figure.
- Define the signal z(n) that is a cosine with frequency 500 Hz, same duration of y(n).
- Define the signal w(n) = z(n) + y(n).

2) [6 pt] Pass the signal w(n) through the following linear systems:
   a) A notch filter to delete the frequency contribution due to x(n), defining w_a(n).
   b) An FIR stop-band filter to attenuate the frequency component due to x(n), defining w_b(n). Set the number of filter samples to 129 and use normalized cut-off frequencies that are +-0.05 with respect to the frequency contribution of x(n).
   c) An all-pass filter with the same poles used for the notch filter in (a), defining w_c(n).
3) [2 pt] Compute the DFT of w_a(n), w_b(n) and w_c(n). Plot their magnitudes as a function of normalized frequency axis.
   - Which one of the possible solutions is able to attenuate at most the frequency contribution due to x(n), while keeping the same behaviour in the remaining frequencies? Motivate your answer.

Solutions

Ex.1



$$H(z)=\frac{\frac{1}{4}+z^{-2}}{1+\frac{1}{4}z^{-2}}$$

It is an all-pass filter

The amplitude response is constant, equal to 1 since it is an all-pass filter.

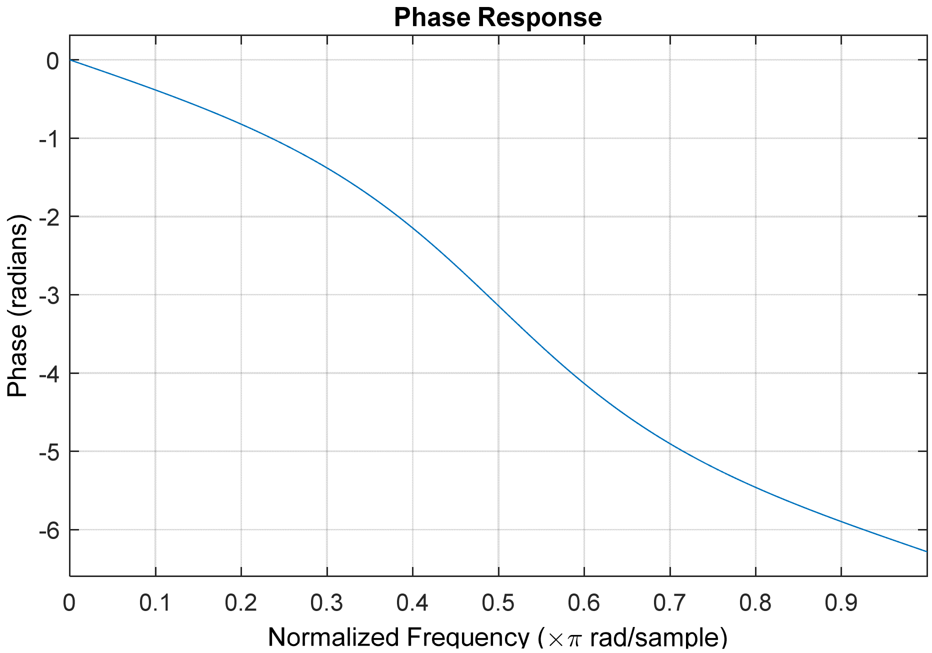
$$\sphericalangle H(\omega)=\sphericalangle\left(\frac{1}{4}+e^{-j2\omega}\right)-\sphericalangle\left(1+\frac{1}{4}e^{-j2\omega}\right)$$

In particolar:

$$\sphericalangle H(\omega=0)=\sphericalangle\left(\frac{1}{4}+1\right)-\sphericalangle\left(1+\frac{1}{4}\right)=0$$

$$\sphericalangle H\left(\omega=\frac{\pi}{2}\right)=\sphericalangle\left(\frac{1}{4}-1\right)-\sphericalangle\left(1-\frac{1}{4}\right)=-\pi$$

$$\sphericalangle H(\omega=\pi)=\sphericalangle\left(\frac{1}{4}+1\right)-\sphericalangle\left(1+\frac{1}{4}\right)=0$$



Concerning the signal $x(n)=1+\cos\left(\frac{\pi}{2}n\right)+\cos(\pi n)$, the output of the filter will remove the central component (since it will be summed to itself with opposite phase) and will double the continuous component and the component at the Nyquist frequency:

$$y(n)=2+2\cos(\pi n)$$

Ex.2

The signal is periodic and each period is $x[n]=\{2,0,-1,0\}$ then its DFT will be $X[k]=\{1,3,1,3\}$.

The filter has to remove the components at the two values: $\omega=\pm\frac{\pi}{2}$ that corresponds to a filter in the DFT: $H(k)=\{1,0,1,0\}$. The filtered signal will then be: $Y[k]=H[k]\cdot X[k]=\{1,0,1,0\}$ and the inverse DFT is: $y[n]=\left\{\frac{1}{2},0,\frac{1}{2},0\right\}$.

Ex.3

```
close all

clearvars

clc


%% 1. [3 pt]


% In a system working at Fs = 2 KHz, you have the signal y(n) (500 samples)

% with the following DFT magnitude (see exam).
% The signal y(n) is the result of zero-padding applied to a cosinusoidal
% signal x(n).
% Define the signal y(n) starting from x(n) followed by zero-padding.
% Hint: which is the original length of x(n)? It can be derived from the figure.
% Define the signal z(n) that is a cosine with frequency 500 Hz,
% same duration of y(n).
% Define the signal w(n) = z(n) + y(n).

% Zero-padding introduces the convolution with a periodic sinc in frequency
% domain.
% The normalized frequency of x(n) is 50/500 (= peak position/number of
% frequency samples).
% The sinc has the first zero in sample = 60.
% --> the original length of x is related to the relative position of the
% first zero with respect to the peak. The length is the inverse of the
% difference between the normalized frequency at sample 60 and that of the
% peak --> length_x = 1/(60/500 - 50/500) = 50 samples.

fx_norm = 0.1;
N_x = 50;
n_x = 0:N_x -1;
x = cos(2*pi*fx_norm*n_x);

% zero-padding up to N = 500;
N = 500;
y = [x, zeros(1, N - N_x)];

% you can check the DFT behaviour (not required)
Y = fft(y);
figure;
plot(0:N-1, abs(Y)./max(abs(Y)), 'LineWidth', 2);
% normalized to adjust the max amplitude
title('|Y(f)|')
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```
xlabel('Frequency samples');
grid;

% define z(n)
fz = 500;
Fs = 2e3;
fz_norm = fz/Fs;
n_z = 0:N-1;
z = cos(2*pi*fz_norm*n_z);

% define w(n)
w = z + y;

%% 2. [6 pt]

% Pass the signal w(n) through the following linear systems:
% a)         A notch filter to delete the frequency contribution due to x(n),
% defining w_a(n).
% b)         An FIR stop-band filter to attenuate the frequency component due
% to x(n), defining w_b(n). Set the number of filter samples to 129,
% and use normalized cut-off frequencies that are +-0.05 with respect
% to the frequency contribution of x(n).
% c)         An all-pass filter with the same poles used for the notch filter
% in (a), defining w_c(n).

% a) notch filter --> poles and zeros in f = +-fx_norm
zeroes = [exp(1i*2*pi*fx_norm); exp(-1i*2*pi*fx_norm)];
poles = zeroes*0.95;
A = poly(poles);
B = poly(zeroes);
w_a = filter(B, A, w);

% filter behaviour (not required)
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[H, omega] = freqz(B, A, 1024, 'whole');
figure,
plot(omega./(2*pi), abs(H), 'LineWidth', 2);
title('|DTFT| of the filter H_{a}(z)');
grid;
xlabel('f [norm]');

% b) FIR stop-band filter
N_filter = 129;
% filter order
filter_order = N_filter - 1;
cutoffs = [fx_norm - 0.05, fx_norm + 0.05];
% matlab cutoff
cutoffs_matlab = cutoffs * 2;
h = fir1(filter_order, cutoffs_matlab, 'stop');
w_b = filter(h, 1, w);

% filter behaviour (not required)
[H, omega] = freqz(h, 1, 1024, 'whole');
figure,
plot(omega./(2*pi), abs(H), 'LineWidth', 2);
title('|DTFT| of the filter H_{b}(z)');
grid;
xlabel('f [norm]');

% c) All-pass filter with the same poles used in (a)
poles = 0.95*[exp(1i*2*pi*fx_norm); exp(-1i*2*pi*fx_norm)];
zeroes = 1./conj(poles);
B = poly(zeroes);
A = poly(poles);
% adjust the gain
c0 = sum(A)/sum(B);
B = B * c0;
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```
w_c = filter(B, A, w);

% filter behaviour (not required)
[H, omega] = freqz(B, A, 1024, 'whole');
figure,
plot(omega./(2*pi), abs(H), 'LineWidth', 2);
title('|DTFT| of the filter H_{c}(z)');
grid;
xlabel('f [norm]');

%% 3. [2 pt]

% Compute the DFT of w_a(n), w_b(n) and w_c(n). Plot their magnitudes
% as a function of normalized frequency axis.
% Which one of the possible solutions is able to attenuate at most the
% frequency contribution due to x(n), while keeping the same behaviour
% in the remaining frequencies? Motivate your answer.

W_a = fft(w_a);
W_b = fft(w_b);
W_c = fft(w_c);

freq_axis = 0:1/N:1 - 1/N;

figure;
semilogy(freq_axis, abs(W_a), 'LineWidth', 2);
hold on;
semilogy(freq_axis, abs(W_b), 'LineWidth', 2, 'LineStyle','--');
semilogy(freq_axis, abs(W_c), 'LineWidth', 2, 'LineStyle','-.');
grid;
xlabel('f [norm]');
legend('|W_a(f)|', '|W_b(f)|', '|W_c(f)|');
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% the best solution is (a), as the filter is a notch, thus it preserves the

% behaviour of the signal in the remaining frequency contributions.
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