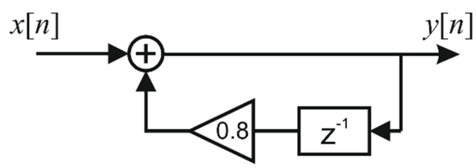
Multimedia Signal Processing 1st Module and Fundamentals of Multimedia Signal Processing

date: January 25th, 2024

Ex.1 (Pt.12)

We have to analyze the following digital filter:



[2pts.] Provide the z-transform of the filter and describe the filter (e.g. the kind of the filter, stability, max, mixed or minimum phase…)

[4 pts.] Provide its pole-zero plot, depict its amplitude and phase behavior at different frequencies

[4 pts.] Design a new FIR filter that approximate the behavior of the pole of the previous filter with 3 zeros: what is its z-transform? Provide its zero-pole plot.

[2 pts.] Describe and represent approximately the differences of the two filters in the amplitude response.

Ex.2 (Pt.10)

Given a signal sampled at 48kHz, we want to downsample it to 32kHz.

[5 pts.] Provide a detailed description of the procedure to change the signal sample rate detailing the parameters of the adopted filters.

[3 pts.] Design a filter with at least 1 pole and 1 zero (outside from the origin) that could be used in the task described above.

[2 pts.] Discuss the impact of the designed filter, with particular reference to the passband, transition band, and stopband, on the utilized signal compared to an ideal filter. Propose a possible measure of the introduced deviation.

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Ex.3 (Pt.12) To be solved writing the MATLAB code on the sheet.

1) [3 pt] You are given two periodic signals, x(n) and y(n):
   - The first samples of x(n) are $[1, \frac{\sqrt{3}}{2}, 0.5, 0, -0.5, -\frac{\sqrt{3}}{2}, -1, -\frac{\sqrt{3}}{2}, -0.5, 0, 0.5, \frac{\sqrt{3}}{2}, 1, \frac{\sqrt{3}}{2}, 0.5]$.
   - The first samples of y(n) are [1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1].
     One period of the signal x(n) is contained in the reported sample set, and the same is for y(n). Define the signal z(n) = x(n) + y(n), which is periodic with period 2.4 milliseconds and has a duration of 12 milliseconds.
2) [4 pt] We want to increase the sampling rate by a factor 1.5. To do so, we have available a filter H(z) defined by this linear finite-difference equation:

   output(n) = input(n) – input(n-12) + output(n-1).

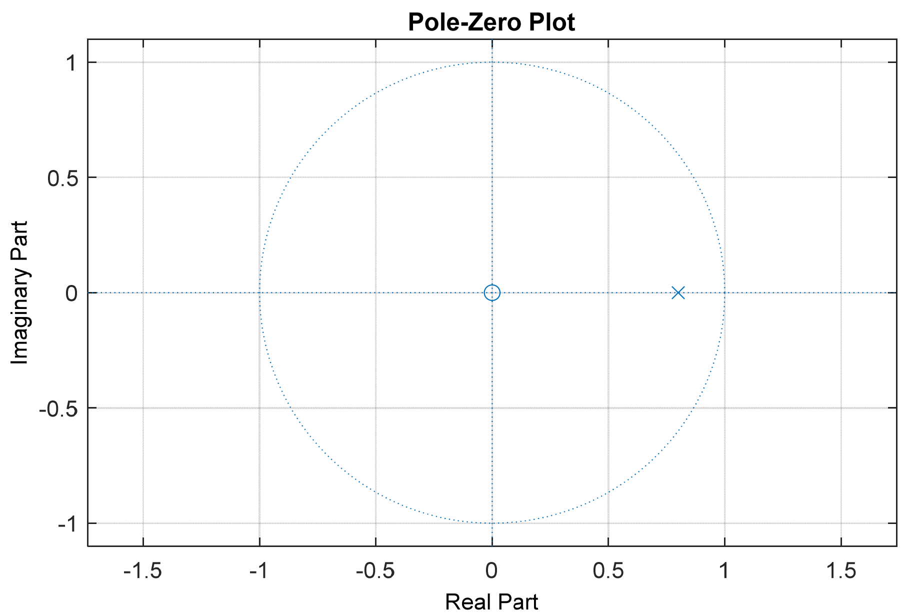
   Answer to the following questions, including motivations:
   - Is the filter stable? (hint: is it FIR or IIR?)
   - Plot the absolute value of the frequency response of the filter vs the normalized frequency axis, considering 2048 samples.
   - Which is the expected behaviour for this filter? (i.e., which is the function associated with it, if it is a low pass or a high pass).
   - Which is the frequency position of the first zero of the filter?
3) [2 pt] Implement the sampling rate conversion of z(n) by using the filter H(z). Define the final signal as z_1(n).
4) [3 pt] Compute the DFT of z_1(n) over 2048 samples. Plot the absolute value of the DFT versus the normalized frequency axis.
   - Which are the theoretical positions (due to the sampling rate conversion) for the peaks of z_1(n)?
   - Given the behaviour of H(z), do you expect to find all the peaks of z_1(n)? Which is the frequency contribution that is kept by the filter? Motivate your answers.

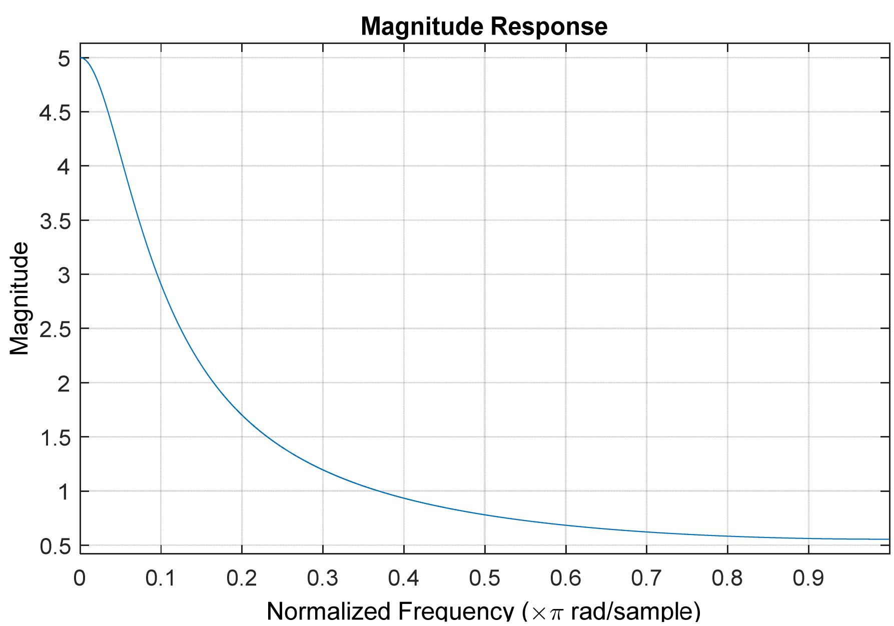
Solutions

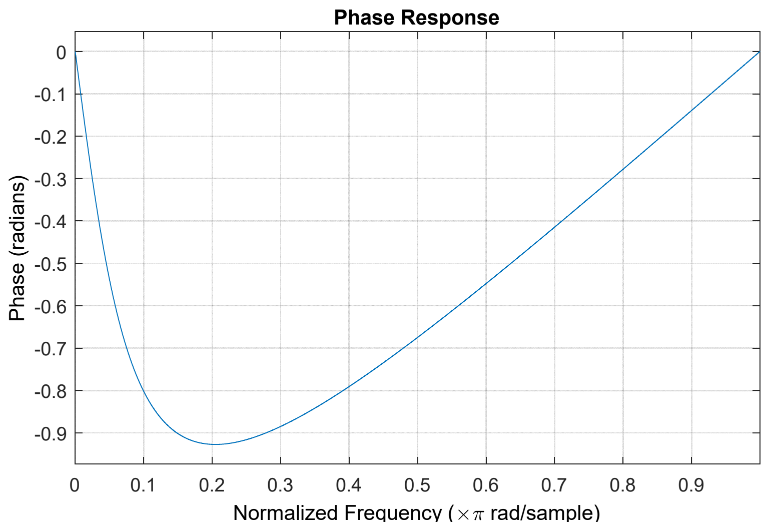
Ex.1

$$H(z) = \frac{1}{1 - 0.8z^{-1}}$$

It is an IIR high-pass stable filter with minimum phase.







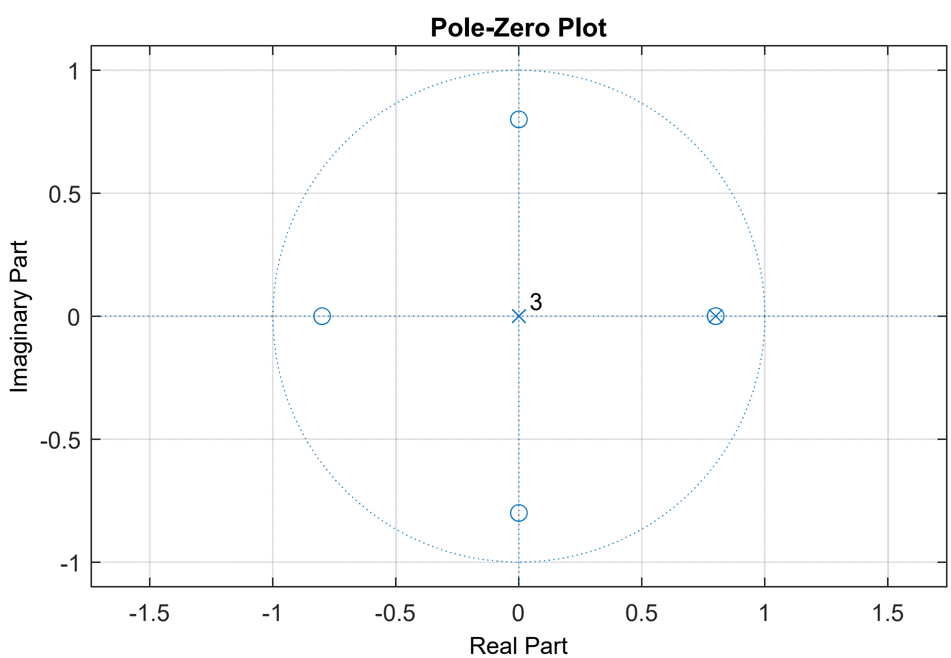
The filter transfer function $H(z)$ can be written as:

$$H(z) = \frac{1}{1-0.8z^{-1}} = \sum_{n=0}^{\infty}\left(0.8z^{-1}\right)^n$$

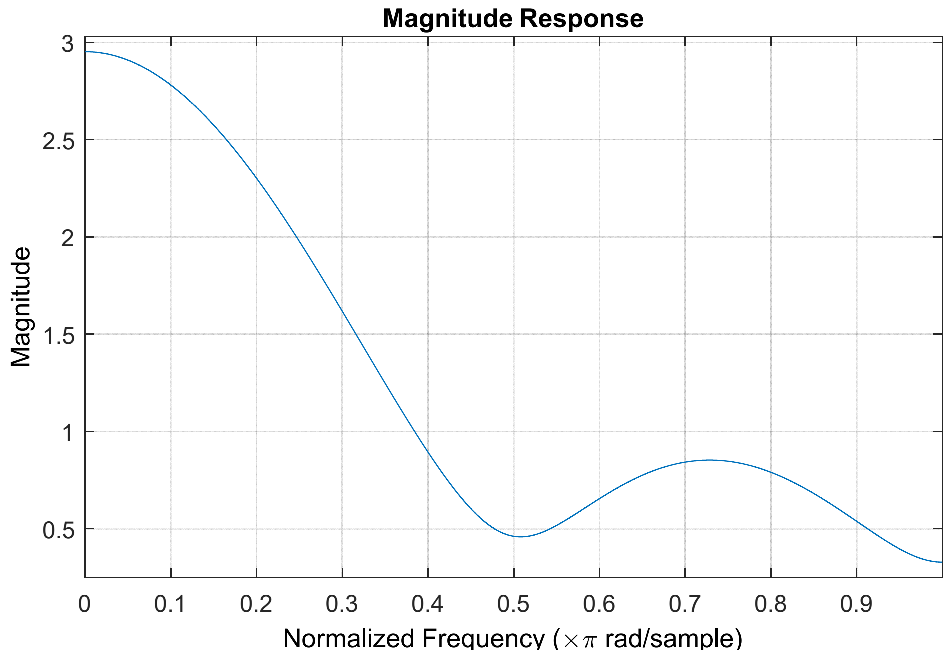
Truncating it to the first 4 terms will give:

$$H_{FIR}(z) = \sum_{n=0}^{3}\left(0.8z^{-1}\right)^n = \frac{1-\left(0.8z^{-1}\right)^4}{1-0.8z^{-1}} = 1 + 0.8z^{-1} + 0.8^2 z^{-2} + 0.8^3 z^{-3}$$

Its zero-pole plot is:



Its amplitude response is:



Ex.2

In order to change the sample rate from 48kHz to 32kHz (2/3 of the original frequency) we need to upsample of an order of 2 (L=2), filter the output with an ideal low pass filter with a cutoff of $\omega_c = \frac{\pi}{3}$ and, then donwsample of an order of 3 (M=3).

Ex.3 (MATLAB CODE)

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close all
clearvars
clc

%% 1. [3 pt]

% You are given two periodic signals, x(n) and y(n):
% The first samples of x(n) are
% [1,  √3/2,0.5,0,-0.5,-√3/2,-1,-√3/2,-0.5,0,0.5,√3/2,1,√3/2,0.5].
% The first samples of y(n) are
% [1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1].
% One period of the signal x(n) is contained in the reported sample
set,
% and the same is for y(n).
% Define the signal z(n) = x(n) + y(n), which is periodic
% with period 2.4 milliseconds and has a duration of 12
milliseconds.

% The first signal has a period = 12. It is a cosine with normalized
% frequency = 1/12 --> x(n) = cos(pi/6*n).
% The second signal has a period = 4. It is a cosine with normalized
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% frequency = 1/4 --> y(n) = cos(pi/2*n).

P_1_sample = 12;
P_2_sample = 4;

% The period of z(n) is the lcm(12, 4) = 12.
% We can find the sampling rate Fs = P_sample / P_sec
P_sample = 12;
P_sec = 2.4e-3;
Fs = P_sample/P_sec;

% frequencies [Hz]
f_1 = 1/P_1_sample * Fs;
f_2 = 1/P_2_sample * Fs;

% time axis [sec]
dur = 12e-3;
t_axis = 0:1/Fs:dur;

% signal definition
z = cos(2*pi*f_1*t_axis) + cos(2*pi*f_2*t_axis);

%% 2. [4 pt]

% We want to increase the sampling rate by a factor 1.5.
% To do so, we have available a filter H(z) defined by this
% linear finite-difference equation:
% output(n) = input(n) - input(n-12) + output(n-1).
% Answer to the following questions, including motivations:
% Is the filter stable? (hint: is it FIR or IIR?)
% Plot the absolute value of the frequency response of the
% filter vs the normalized frequency axis, considering 2048 samples.
% Which is the expected behaviour for this filter
% (the function associated with it, if it is a low pass or a high
pass).
% Which is the frequency position of the first zero of the filter?

% numerator (we need to insert 11 zeros in the middle to account for
all
% the zero coefficients from z^{-1} up to z^{-11}
B = [1, zeros(1, 11), -1];
% denominator
A = [1, -1];

% the filter is stable.
% H(z) = 1/ (1 - z^(-1)) - z^(-12) / (1 - z^(-1))
% --> h(n) = u(n) - u(n-12)
% the second term deletes the first one for n >= 12.
% this is a rectangular window with size (number of samples
different from
% 0) = 12.
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% --> the filter is FIR, therefore it is stable.

% visualize the time domain behaviour of the filter (not required)
% first 30 samples
h = filter(B, A, [1, zeros(1, 29)]);
figure,
stem(0:29, h);
grid;
title('h(n)');
grid;
xlabel('n');

% frequency response of the filter
N = 2048;
[H, omega] = freqz(B, A, N, 'whole');
figure;
plot(omega./(2*pi), abs(H));
title('|DTFT| of the filter H(z)');
grid;
xlabel('f [norm]');

% the expected behaviour is that of a sinc function, since the
% filter has a rectangular behaviour in the time.
% The first zero of this filter can be found at normalized frequency
=
% 1/rectangular_window_size = 1/12.
% Therefore, the filter is a low pass filter.

%% 3. [2 pt]

% Implement the sampling rate conversion of z(n) by using the filter
H(z).
% Define the final signal as z_1(n).

% since we need to change the sampling rate by a factor 1.5
L = 3;
M = 2;

% first, upsampling
z_upsampled = zeros(1, length(z) * L);
z_upsampled(1:L:end) = z;

% filtering
z_f = filter(B, A, z_upsampled);

% decimation
z_1 = z_f(1:M:end);

%% 4. [2 pt]

% Compute the DFT of z_1(n) over 2048 samples.
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% Plot the absolute value of the DFT versus the normalized frequency
axis.
% Which are the theoretical positions (due to the sampling rate
conversion)
% for the peaks of z_1(n)?
% Given the behaviour of H(z), do you expect to find all the peaks
of z_1(n)?
% Which is the frequency contribution that is kept by the filter?
% Motivate your answers.

N = 2048;
Z_1 = fft(z_1, N);

% normalized frequency axis:
freq_axis = 0:1/N:1 - 1/N;

% absolute value of dft(z(n)) (not required)
Z = fft(z, N);
figure;
stem(freq_axis, abs(Z));
title('Absolute value of the DFT of the signal z(n)');
grid;
xlabel('f [norm]');
% There are two peaks at f_1 = 1/12 and f_2 = 1/4, together with
their
% symmetric components.

% absolute value of Z_1(f)
figure;
stem(freq_axis, abs(Z_1));
title('Absolute value of the DFT of the signal z_1(n)');
grid;
xlabel('f [norm]');

% theoretical positions of the peaks:
% when upsampling by L = 3, the peaks of z(n) are moved in 1/36 (due
to f_1)
% and 1/12 (due to f_2), then in 1/3 - 1/36 and 1/3 - 1/12,
% plus all the other replicas...
% when filtering, we expect that everything after the cutoff (= 1/6)
is
% removed by the filter --> only the peaks at 1/36 and 1/12 should
remain.
% when downsampling by M = 2, the peaks are moved in 1/18 (due to
f_1)
% and 1/6 (due to f_2) with their related symmetric.

% Given this specific H(z), we don't expect to find all the
theoretical
% peaks in the final result. Indeed, H(z) has a sinc behaviour and
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% it has zeros every 1/12 in the normalized frequency domain. Since
the
% first zero is exactly in 1/12, this strongly attenuates the
frequency
% component f_2 (after the upsampling, f_2 has been moved to 1/12).
% Therefore, in the final result, we expect to find only two main
peaks:
% one in 1/18 (due to f_1) and its related symmetric.
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