Multimedia Signal Processing 1st Module and Fundamentals of Multimedia Signal Processing

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Ex.1 (Pt.11)

A signal $x[n]$ is filtered using a filter $h[n]$ that is the cascade of two filters, $h_1[n]=\{1,-1\}$ and $h_2[n]=\{1,0,4\}$.

1. [3 pts] Find the temporal sequence of the filter $h[n]=\ldots$ and its z-transform $H(z)=\ldots$
2. [3 pts] Represent the pole-zero plot of $H(z)$ and its magnitude.
3. [2 pts] In case of it is a maximum phase filter provide its minimum phase version with the same magnitude response otherwise, if it is a "minimum phase filter" provide its maximum phase version $\bar{H}(z)$.
4. [3 pts] Working in the time domain evaluate the output $y[n]=\ldots$ of the signals $x_a[n]=\{1,-1,1,-1\}$ and $x_b[n]=\{1,1,1,1\}$ filtered with the filter $h[n]$.

Ex.2 (Pt.10)

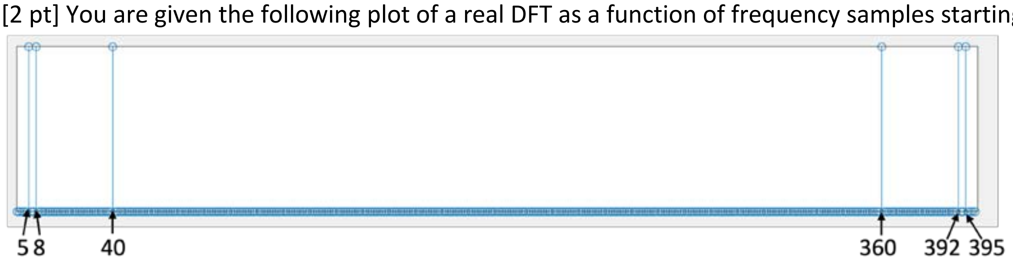
A signal $x(t)=\sin(2\pi 100t)+2\cos(2\pi 50t)$ is sampled at 400Hz and the downsampled by an order of $M=3$.

1. [5 pts] What is the output in case of no low pass (antialiasing) filter is adopted? Depict the magnitude of the output in the range of $0-\pi$ (normalized frequencies).
2. [5pts] Suggest a possible low pass filter describing its behavior and the cut-off frequency and justifying the choice: what will be the effect on the output signal?

Ex.3 (Pt.12)

To be solved with MATLAB and to be uploaded on Webeep assignments folder

1) [2 pt] You are given the following plot of a real DFT as a function of frequency samples starting from 0:



Define the discrete-time signal x(n) related to this DFT. (hint: the amplitude of each signal component is equal to 1, and the DFT has been computed using the total amount of signal samples (400), without zero-padding).

2) [3 pt] You are given a set of zeroes and a set of poles as follows:

- Zeroes can be selected from this set: [1/1.01*exp(pi*1i/5), 0.9*exp(-pi*1i/40), 1/5*exp(pi*1i/40), ... 1/1.01*exp(-pi*1i/5), 1/1.1*exp(-pi*1i/25), 1/5*exp(-pi*1i/40), 1.01*exp(pi*1i/40), 1.01*exp(-pi*1i/40),... 1/1.1*exp(pi*1i/25)]

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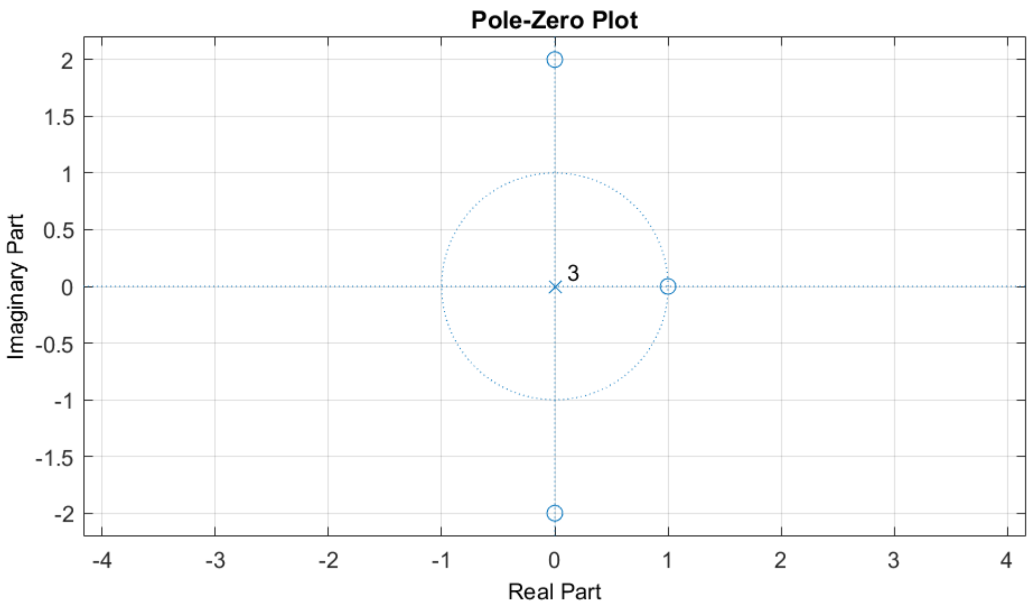
- Poles can be selected from this set: [0.99*exp(pi*1i/25), 1.01*exp(pi*1i/25), 0.99*exp(-pi*1i/5), ... 0.99*exp(-pi*1i/25), 1.01*exp(pi*1i/40), 0.9, 0.99*exp(pi*1i/5)];
- Choose the poles and the zeroes to build a minimum phase filter H_min(z) with real coefficients such that, when filtering the signal x, we can attenuate the lowest and the highest sinusoidal component of x, and we can enhance the mid sinusoidal component of x. Set the filter gain (in frequency 0) to 0.4.
- Verify that the chosen filter is minimum phase by plotting its zeros and poles in the complex plane.

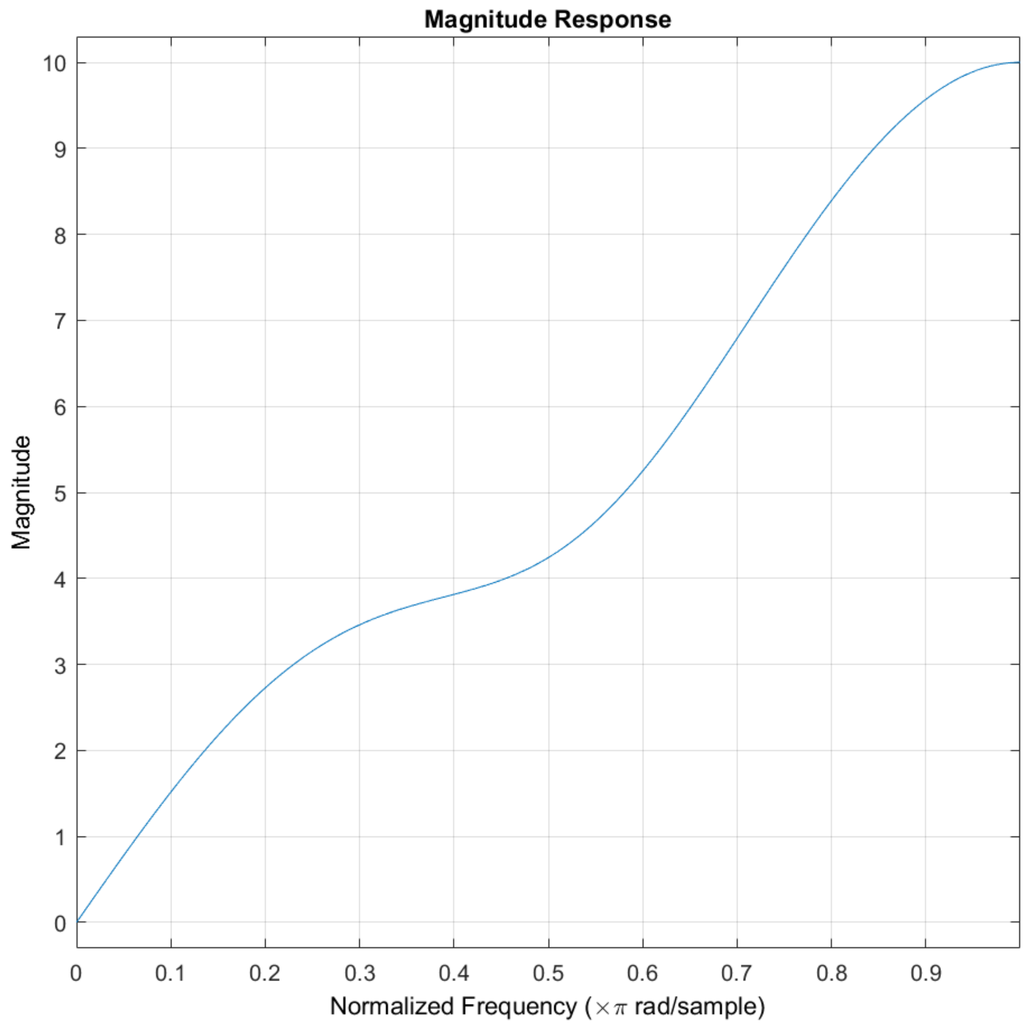
3) [4 pt] Find the causal and stable filter H(z) such that H_min(z) is the minimum-phase component of the allpass-minimum-phase decomposition of H(z). We know that H(z) has some zeroes with absolute value = 5.
   - Which should be the gain (in frequency 0) of the filter H(z)?
   - To find H(z), do we need to also find the all-pass component?
   - Find the all-pass component of H(z), defining it as H_ap(z).
   - Verify that H(z) = H_min(z) * H_ap(z), exploiting the convolution property and, by using the function 'freqz' over 1024 samples, plotting the absolute value of the frequency response of H(z) and of the filter found as H_min(z) * H_ap(z) over the entire frequency spectrum in normalized domain.
4) [3 pt] Filter the signal x with the filters H_min(z), H_ap(z) and H(z) and plot the absolute value of the DFT of the filtered signals as a function of normalized frequency.
   - In which cases do we have a DFT different or similar to that of the original signal x?
   - In some cases, we have some sinusoidal components that have been attenuated. Which are these components? Which is the most attenuated one? Motivate your answer.
   - Which are the differences between the signal filtered with H_min(z) and the signal filtered with H(z)?

Solutions

Ex.1

$h[n] = h_1[n] * h_2[n] = \{1, -1, 4, -4\}$, $H(z) = 1 - z^{-1} + 4z^{-2} - 4z^{-3}$





The filter is a maximum phase filter in order to get a minimum phase with the same magnitude we have to set a gain and move the zeros inside in reciprocal positions:

$$\bar{H}(z) = G\left(1 - z^{-1} + \frac{z^{-2}}{4} - \frac{z^{-3}}{4}\right)$$

Forcing, e.g., $H(z=-1) = \bar{H}(z=-1)$ we get $G = 4$

$y_a = [1 \quad -2 \quad 6 \quad -10 \quad 9 \quad -8 \quad 4]$

$y_b = [1 \quad 0 \quad 4 \quad 0 \quad -1 \quad 0 \quad -4]$

Ex.2

Before sampling, in the range of normalized frequencies ($0-\pi$), we will have:

$$|X(\omega)| = \delta\left(\omega - \frac{\pi}{4}\right) + \frac{1}{2}\delta\left(\omega - \frac{\pi}{2}\right)$$

After downsampling without low pass filters the impulse will shift to:

$$\delta\left(\omega-\frac{\pi}{4}\right)\rightarrow\delta\left(\omega-\frac{3\pi}{4}\right) \text{ and } \delta\left(\omega-\frac{\pi}{2}\right)\rightarrow\delta\left(\omega-\frac{3\pi}{2}\right)$$

But, due to the replicas of downsampling, we will also have, in the range ($0-\pi$), an impulse in $2\pi-\frac{3\pi}{2}$. The magnitude of the spectrum will then be:

$$|Xd(\omega)|=\frac{1}{3}\left[\delta\left(\omega-\frac{3\pi}{4}\right)+\frac{1}{2}\delta\left(\omega-\left(2\pi-\frac{3}{2}\pi\right)\right)\right]=$$

$$=\frac{1}{3}\left[\frac{1}{2}\delta\left(\omega-\frac{\pi}{2}\right)+\delta\left(\omega-\frac{3\pi}{4}\right)\right]$$

$$\omega=\frac{\pi}{3}$$

With an ideal low pass the aliased components will not be present thanks to a low pass filter with a cutoff frequency of $\omega=\frac{\pi}{3}$

Ex.3 (MATLAB CODE)

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close all
clearvars
clc

%% 1.

% [2 pt] You are given the following plot of a real DFT as a
% function of frequency samples starting from 0:
% (see the exam text).
% Define the discrete-time signal x related to this DFT.
% (hint: the amplitude of each signal component is equal to 1, and the
% DFT has been computed using the total amount of signal samples (400),
% without zero-padding).

N = 400;
% the signal is composed by 3 cosinusoids, as the DFT is real and we find 6
% peaks (3 peaks before N/2, and other 3 peaks mirrored after N/2).
% the normalized frequencies correspond to the peak positions divided by N =
400.
f_vec = [5, 8, 40]./N;
f0 = f_vec(1);
f1 = f_vec(2);
f2 = f_vec(3);

% define the signal x:
n = 0:N-1;
x = sum(cos(2*pi*f_vec'.*n), 1);

%% 2.
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% [3 pt] You are given a set of zeroes and a set of poles as follows:

zeroes_set = [1/1.01*exp(pi*1i/5), 0.9*exp(-pi*1i/40), 1/5*exp(pi*1i/40), ...
1/1.01*exp(-pi*1i/5), 1/1.1*exp(-pi*1i/25), 1/5*exp(-pi*1i/40),
1.01*exp(pi*1i/40), ...
1.01*exp(-pi*1i/40), 1/1.1*exp(pi*1i/25)];

poles_set = [0.99*exp(pi*1i/25), 1.01*exp(pi*1i/25), 0.99*exp(-pi*1i/5), ...
0.99*exp(-pi*1i/25), 1.01*exp(pi*1i/40), 0.9, 0.99*exp(pi*1i/5)];

% Choose the poles and the zeroes to build a minimum phase filter H_min(z)
% with real coefficients such that, when filtering the signal x,
% we can attenuate the lowest and the highest sinusoidal component of x,
% and we can enhance the mid sinusoidal component of x.
% Set the filter gain (in frequency 0) to 0.4.
% Verify that the chosen filter is minimum phase by plotting its
% zeros and poles in the complex plane.
% Verify the filter behaviour by using the function freqz over 1024
% samples, plotting the absolute value of the frequency response of H_min(z)
% over the entire frequency spectrum in normalized domain.

% keep only the zeros and poles which are inside the unit circle and such
that the
% resulting coefficients are real. f0 and f2 should be attenuated, f1
% should be enhanced.
% f0 = 5/400 --> 1/80 --> omega_0 = pi/40
% f1 = 8/400 --> 1/50 --> omega_1 = pi/25
% f2 = 40/400 --> 1/10 --> omega_1 = pi/5

zeroes = [1/1.01*exp(pi*1i/5), 1/1.01*exp(-pi*1i/5), 1/5*exp(pi*1i/40),
1/5*exp(-pi*1i/40)];
poles = [0.99*exp(pi*1i/25), 0.99*exp(-pi*1i/25)];

b_min = poly(zeroes);
a_min = poly(poles);

% find the numerator and denominator of the filter H_min such that
% the filter gain in frequency = 0 is equal to 0.4.
% In order to have gain = 0.4 in f = 0, we should impose that:
% k*B_min(z=1)/A_min(z=1) = 0.4 --> k = 0.8*sum(a_min)/sum(b_min).
k = 0.4*sum(a_min) / sum(b_min);
% multiply the numerator by k
b_min = k * b_min;

% Verify that the chosen filter is minimum phase by plotting its
% zeros and poles in the complex plane.

figure;
zplane(b_min, a_min);

%% 3.

% [3 pt] Find the causal and stable filter H(z) such that H_min(z)
% is the minimum-phase component of the allpass-minimum-phase decomposition
% of H(z). We know that H(z) has some zeroes with absolute value = 5
% Which should be the gain (in frequency 0) of the filter H(z)?
% To find H(z), do we need to also find the all-pass component?
% Find the all-pass component of H(z), defining it as H_ap(z).
% Verify that H(z) = H_min(z) * H_ap(z), exploiting the convolution property
% and, by using the function freqz over 1024 samples,
% plotting the absolute value of the frequency response of H(z) and of
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% the filter found as H_min(z) * H_ap(z) over the entire frequency spectrum
% in normalized domain.

% The zeroes and poles selected for the minimum phase filter were:
% zeroes = [1/1.01*exp(pi*1i/5), 1/1.01*exp(-pi*1i/5), 1/5*exp(pi*1i/40),
% 1/5*exp(-pi*1i/40)];
% poles = [0.99*exp(pi*1i/25), 0.99*exp(-pi*1i/25)];

% By definition, the minimum-phase component contains:
% the poles and zeros of H(z) that lie inside the unit circle
% zeros that are conjugate reciprocals of the zeros of H(z) lying outside the
% unit circle.

% --> Since we know that H(z) has some zeroes with absolute value = 5,
% the only original minimum phase zeroes of the filter H(z) must be
% 1/1.01*exp(pi*1i/5)and 1/1.01*exp(-pi*1i/5). The other zeros will have
% absolute value = 5 and phase equal to pi/40.
% Since H(z) is stable and causal, the poles were all inside the unit
% circle. Thus, the poles of H(z) are those of H_min(z).

zeroes_H = [1/1.01*exp(pi*1i/5), 1/1.01*exp(-pi*1i/5), 5*exp(pi*1i/40), ...
    5*exp(-pi*1i/40)];
poles_H = [0.99*exp(pi*1i/25), 0.99*exp(-pi*1i/25)];

b_H = poly(zeroes_H);
a_H = poly(poles_H);

% Which should be the gain (in frequency 0) of the filter H(z)?
% the gain is (for sure) the same of the minimum-phase one

k = 0.4*sum(a_H) / sum(b_H);
% multiply the numerator by k
b_H = k * b_H;

% To find H(z), do we need to also find the all-pass component?
% No

% Find the all-pass component of H(z), defining it as H_ap(z).
% By definition, the minimum-phase component contains:
% all the zeros of H(z) that lie outside the unit circle
% poles which are conjugate reciprocals of the zeros of H(z) lying outside
the
% unit circle

% --> the zeroes are only the maximum phase ones, and the poles are their
% conjugate reciprocal
zeroes_ap = [5*exp(pi*1i/40), 5*exp(-pi*1i/40)];
poles_ap = 1./conj(zeroes_ap);

b_ap = poly(zeroes_ap);
a_ap = poly(poles_ap);

% Remember to set the filter gain = 1
k = sum(a_ap) / sum(b_ap);
% multiply the numerator b_ap by k
b_ap = k*b_ap;

% Verify that H(z) = H_min(z) * H_ap(z), exploiting the convolution property
% and, by using the function freqz over 1024 samples,
% plotting the absolute value of the frequency response of H(z) and of
% the filter found as H_min(z) * H_ap(z) over the entire frequency spectrum
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% in normalized domain.

% filter found as H_ap(z) * H_min(z) = H_ap_min(z)
b_ap_min = conv(b_ap, b_min);
a_ap_min = conv(a_ap, a_min);

% frequency responses
M = 1024;
H_ap_min = freqz(b_ap_min, a_ap_min, M, 'whole');
H = freqz(b_H, a_H, M, 'whole');

figure;
semilogy(0:1/M:(1-1/M), abs(H));
leg{1} = 'Filter H(z)';
hold on;
semilogy(0:1/M:(1-1/M), abs(H_ap_min), '--');
leg{2} = 'Filter found as H_ap(z) * H_min(z)';
legend(leg);

%% 4.

% [3 pt] Filter the signal x with the filters H_min(z), H_ap(z) and H(z)
% and plot the absolute value of the DFT of the filtered signals as a
% function of normalized frequency.
% In which cases do we have a DFT different or similar to that of the
% original signal x?
% In some cases, we have some sinusoidal components that have
% been attenuated. Which are these components? Which is the most attenuated
one?
% Motivate your answer.
% Which are the differences between the signal filtered with H_min(z)
% and the signal filtered with H(z)?

x_min = filter(b_min, a_min, x);
x_ap = filter(b_ap, a_ap, x);
x_H = filter(b_H, a_H, x);

X_min = fft(x_min);
X_ap = fft(x_ap);
X_H = fft(x_H);

freq_axis = 0:1/length(X_min):1-1/length(X_min);

figure;
plot(freq_axis, abs(X_min));
leg{1} = 'Minimum-phase filtered signal';
hold on;
plot(freq_axis, abs(X_ap), '*-');
leg{2} = 'All-pass filtered signal';
plot(freq_axis, abs(X_H), 'o-');
leg{3} = 'Signal filtered with H(z)';
legend(leg);

% In which cases do we have a DFT different or similar to that of the
% original signal x?

% Different: X_min and X_H
% Similar: X_ap

% In some cases, we have some sinusoidal components that have
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% been attenuated. Which are these components? Which is the most attenuated
one?
% Motivate your answer.

% In case of X_min and X_H, we have the sinusoidal component in f2 which is
% strongly attenuated, as the absolute value of the zeroes corresponding to
this frequency
% was nearly 1.
% The sinusoidal component in f0 is only slightly attenuated,
% as the zeroes corresponding to f0 were far from the unit circle.

% Which are the differences between the signal filtered with H_min(z)
% and the signal filtered with H(z)?

% From the point of view of the absolute value of the DFTs, there are no
% differences. The two signals differ in their phase content.
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