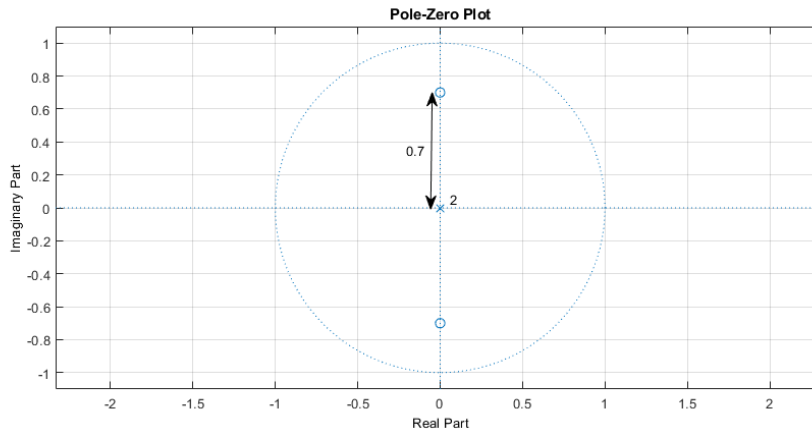


## Multimedia Signal Processing 1<sup>st</sup> Module and Fundamentals of Multimedia Signal Processing

date: 8/11/2021

### Ex.1 (Pt.15)

A digital filter  $H_m(z)$  is a minimum phase filter with the following pole-zero diagram:



1. [3 pts] provide the equation of  $H_m(z)$  in  $z$ .
2. [3 pts] provide an approximate representation of its magnitude and phase response finding the exact values for  $\omega = 0, \omega = \pi/2, \omega = \pi$ .

Transform the filter  $H_m(z)$  into its maximum phase equivalent filter  $H_M(z)$  preserving the same amplitude response.

3. [3 pts] provide the equation of  $H_M(z)$  with the proper gain.

Just displacing the poles transform the filter  $H_M(z)$  into an “all pass” filter  $H_A(z)$ .

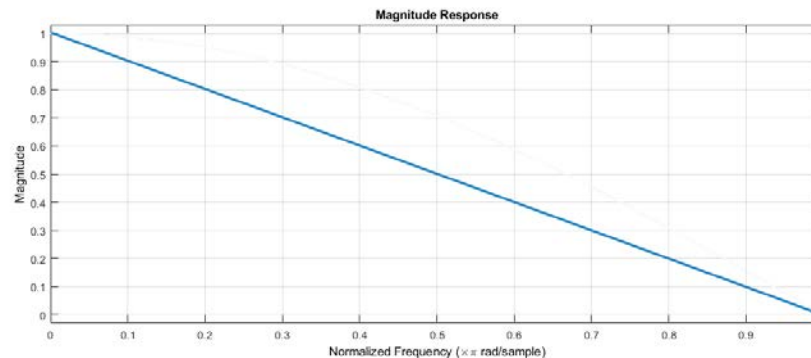
4. [3 pts] provide the equation of  $H_A(z)$ , what will be the effect on the phase at  $\omega = 0, \omega = \pi/2, \omega = \pi$ ?
5. [3 pts] provide the finite difference equation ( $y[n] = \dots$ ) of the  $H_A(z)$  filter.

### Ex.2 (Pt. 8)

A continuous signal composed of two sinusoids  $y(t) = 10 \cos(2\pi \cdot 50t) + 5 \sin(2\pi \cdot 75t)$ . It is sampled at 200Hz with an ideal sampler.

1. [2 pts] we need to change its sample rate to 300 samples per second: describe, to processing chain in order to get the desired result from a quantitative viewpoint (providing the values used in each step).

2. [6 pts] In order to get the result we can simply use a low pass filter whose frequency behavior is depicted in the following graph:



The magnitude decreases linearly from 1 at  $\omega = 0$  to 0 at  $\omega = \pi$  and the phase is null for the whole spectrum.

Describe the output signal underlining the impact (gain/attenuation) of such a filter on the signal components and on the spurious components due to the sample rate change.

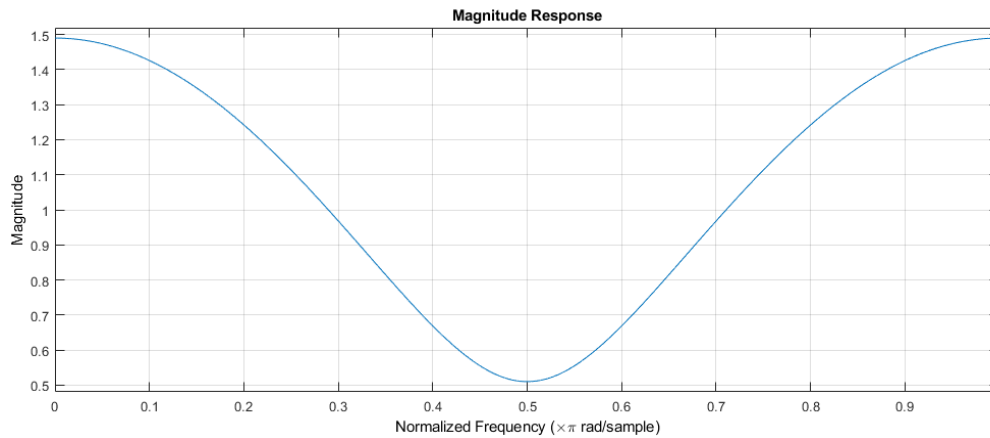
### Ex.3 (Pt. 11) [provide the MATLAB code]

1. [4 pt] The signal  $w(n)$ , sampled with  $F_s = 1\text{KHz}$ , is defined as the sum of three sinusoidal signals, where the first signal has frequency  $f_0$ , the second signal has frequency  $f_0/2$  and the third one has frequency  $f_0/4$ . Define the signal  $w(n)$  such that it repeats periodically every 10 ms. Consider a number of time-samples equal to 10 periods of  $w(n)$ .
2. [4 pt] You are given two zeroes:  $z_1, z_2$ , with absolute value equal to 2, in complex conjugate position. Exploit these two zeroes to build the following filters:
  - a.  $H_a(z)$ , which is an FIR filter (without poles) in order to attenuate the middle frequency of the signal  $w(n)$ ;
  - b.  $H_b(z)$ , which is the all-pass filter related to the filter  $H_a(z)$  (remember to set the gain correctly);
  - c.  $H_c(z)$ , which is the minimum phase filter with the same exact magnitude response of  $H_a(z)$ ;
  - d.  $H_d(z)$ , which attenuates the middle frequency of the signal  $w(n)$  but maintains unaltered the frequency content of the signal in the other frequencies (hint: to build this filter, you can modify the absolute value of  $z_1, z_2$ ).
3. [1 pt] Filter the signal  $w(n)$  with every filter defined in 2), obtaining  $w_a(n), w_b(n), w_c(n), w_d(n)$ .
4. [2 pt] Compute the DFTs of the signals  $w(n), w_a(n), w_b(n), w_c(n), w_d(n)$ , and plot (using the function "stem") their absolute values as a function of the normalized frequency axis, starting from frequency 0.
  - e. What do you expect to see in the DFTs of  $w_a(n), w_b(n), w_c(n), w_d(n)$ , from the point of view of the DFTs amplitude with respect to the original signal  $w(n)$ ? Comment the results for each different signal.
  - f. What do you expect to see in the DFTs of  $w_a(n)$  and  $w_c(n)$ , from the point of view of the DFTs amplitude? And in the phase?

## Solutions

### Ex.1

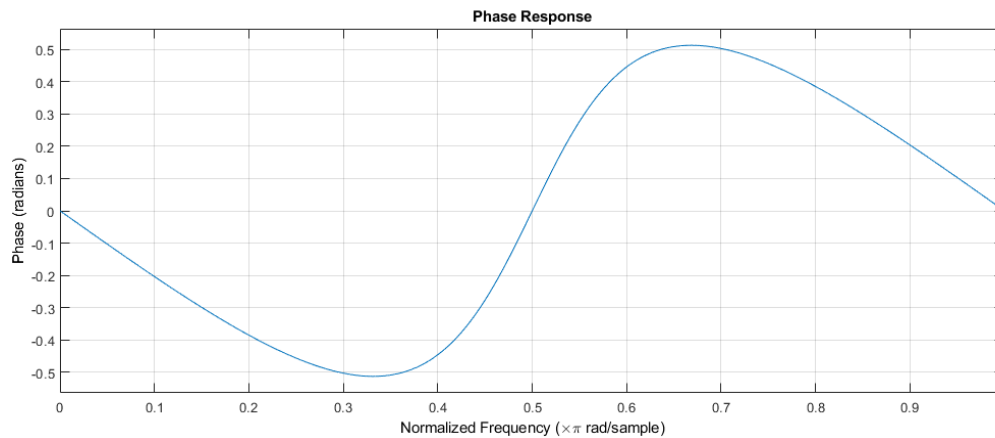
$$H_m(z) = 1 + 0.7^2 z^{-2} = 1 + 0.49z^{-2}$$



$$H_m(z = e^{j0} = 1) = 1.49$$

$$H_m(z = e^{j\pi/2} = j) = 1 - 0.49 = 0.51$$

$$H_m(z = e^{j\pi} = -1) = 1 + 0.49 = 1.49$$



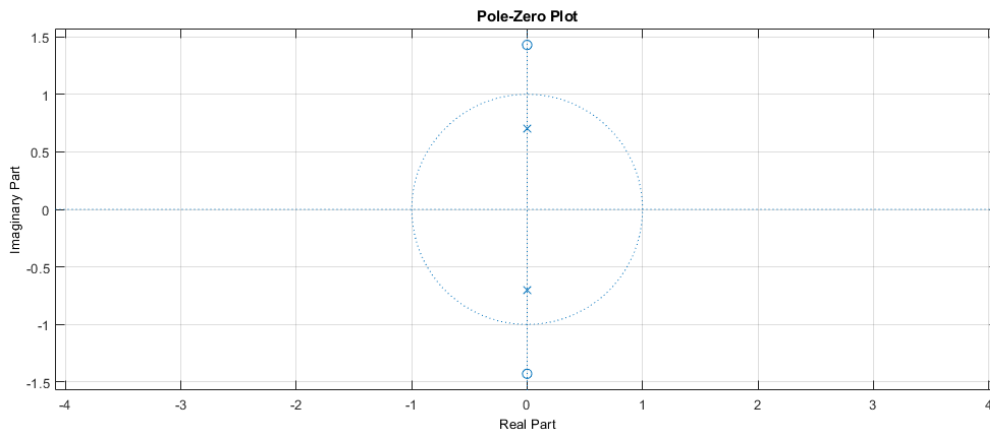
1. The phase in  $\omega = 0, \frac{\pi}{2}, \pi$  will always be zero since the contributions from the zeros is compensated by the contributions of the poles.

For the maximum phase filter the zeros have to go into their reciprocal conjugate position:

$$H_M(z) = G \left( 1 + \left( \frac{1}{0.7} \right)^2 z^{-2} \right)$$

Simply forcing  $|H_M(z)| = |H_m(z)|$  in, e.g.,  $\omega = 0$ , will give  $G \left( 1 + \left( \frac{1}{0.7} \right)^2 \right) = 1.49$  then  $G = 0.49$ .

$$H_A(z) = 0.49 \frac{1 + (0.7)^{-2} z^{-2}}{1 + (0.7)^2 z^{-2}}$$



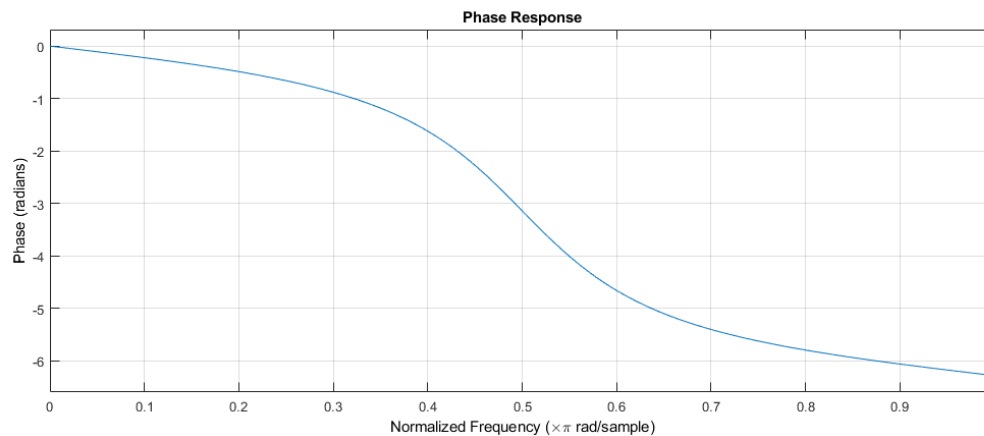
The amplitude response is flat according to all pass filters.

The phase will be the following, in particular:

$$\angle H(\omega = 0) = 0$$

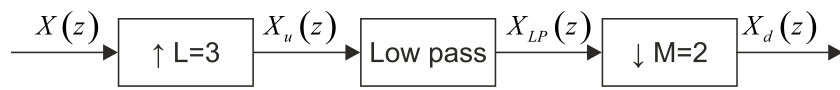
$$\angle H(\omega = \pi/2) = -\pi$$

$$\angle H(\omega = \pi) = -2\pi$$

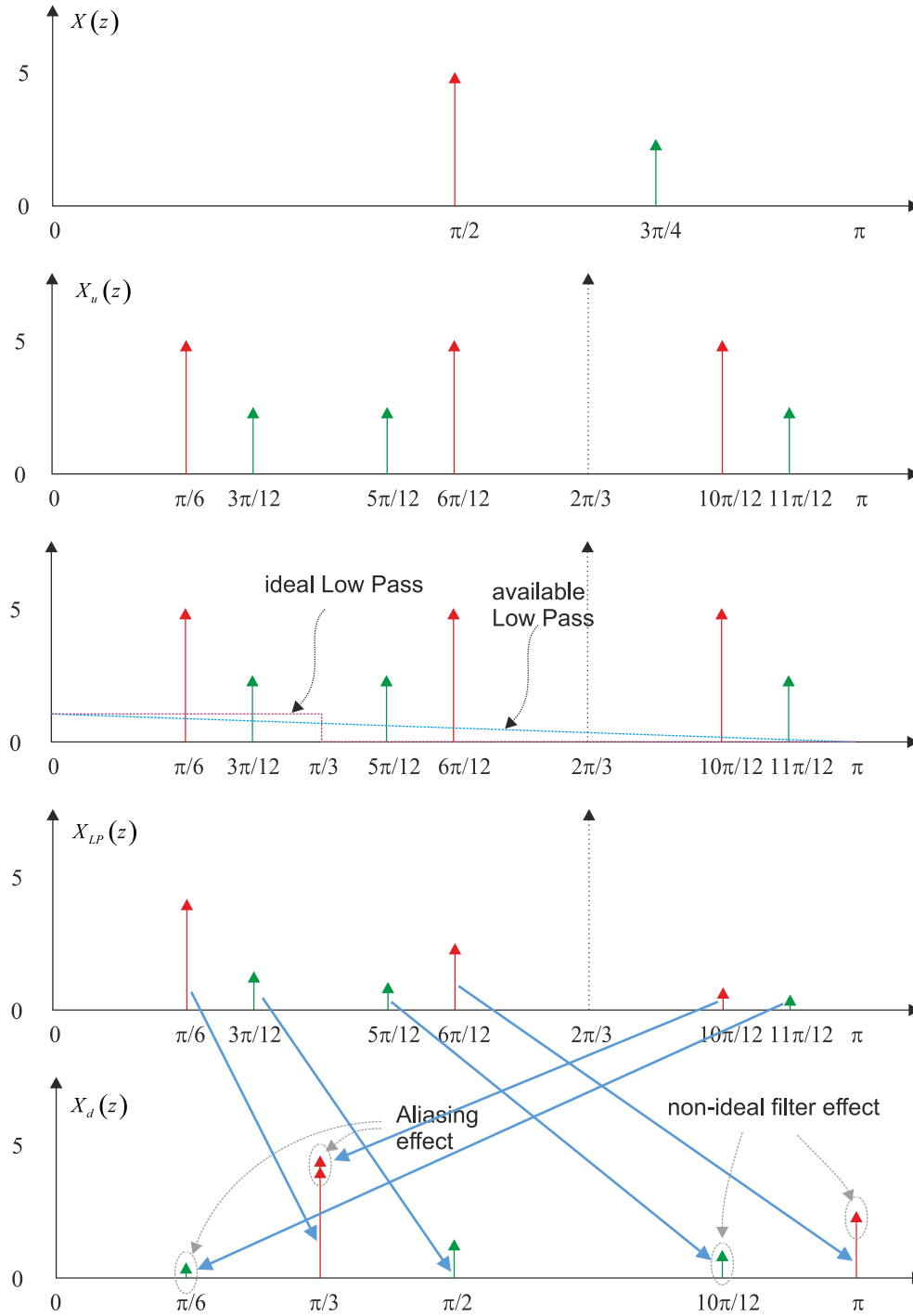


## Ex.2

The sequence is the following:



The spectrum of the signal in the different steps will be:



In the representation above we represented the amplitude of the signal in the different steps. However, according to the representation, the green component can be considered as the real component while the red one represents the imaginary component.

The low pass filter attenuates linearly the components without introducing changes in the amplitudes while the downsampling creates spurious components at  $\frac{\pi}{6}, \frac{\pi}{3}, \frac{10}{12}\pi, \pi$  due to the non-ideal low pass filter, furthermore the first two components at  $\frac{\pi}{6}, \frac{\pi}{3}$  are aliasing components.

### Ex.3

```
close all
clearvars
clc

%% 1.

% [4 pt] The signal w(n), sampled with Fs = 1KHz, is defined as the
sum of
% three cosinusoidal signals, where the first signal has frequency f0,
% the second signal has frequency f0/2 and the third one has frequency
f0/4.
% Define the signal w(n) such that it repeats periodically every 10
ms.
% Consider a number of time-samples equal to 10 periods of w(n).

Fs = 1e3;
P = 0.01;
P_samples = P*Fs;

% find the least common multiple among 1/f0, 2/f0, 4/f0
% --> 4/f0 = P_samples --> f0 = 4/P_samples
f0 = 4/P_samples;
f1 = f0/2;
f2 = f0/4;

N = 10*P_samples;
n = 0:N-1;

w = cos(2*pi*f0*n) + cos(2*pi*f1*n) + cos(2*pi*f2*n);

%% 2.

% [4 pt] You are given two zeroes: z1, z2, with absolute value equal
to 2,
% in complex conjugate position.
% Exploit these two zeroes to build the following filters:
% Ha(z), which is an FIR filter (without poles) in order to attenuate
% the middle frequency of the signal w(n);
% Hb(z), which is the all-pass filter related to the filter Ha(z)
```

```

% (remember to set the gain correctly);
% Hc(z), which is the minimum phase filter with the same exact
% magnitude response of Ha(z);
% Hd(z), which attenuates the middle frequency of the signal w(n) but
% maintains unaltered the frequency content of the signal in the other
% frequencies (hint: to build this filter, you can modify the absolute
% value of z1, z2).

%% Ha

% The middle frequency of w(n) is f1
z1 = 2*exp(1i*2*pi*f1);
z2 = conj(z1);

% actually, Ha = Ba, as Aa will be = 1
Aa = 1;
% Ba can be defined in three different ways:

% 1)if we have two zeroes which are complex conjugate, we can
% always define the polynomial related to the zeros as
%  $1 - 2*\rho*\cos(\theta)z^{-1} + \rho^2 z^{-2}$ 
Ba = [1, -2*2*cos(2*pi*f1), 4];

% 2)exploit the convolution property
% Ba = conv([1 -z1], [1 -z2]);

% 3)define the zeroes and create the polynomial
% zeroes_a = [z1; z2];
% Ba = poly(zeroes_a);

% % to check the zplane and the behaviour of the filter in frequency:
% figure;
% zplane(Ba, Aa);
% title('Z-plane of filter Ha');
% grid;
%
% [Ha, omega] = freqz(Ba, Aa, N, 'whole');
% figure,
% plot(omega./(2*pi), abs(Ha));
% title('DTFT of filter Ha');
% grid;

%% Hb

% can be defined as Ba divided by a polynomial with the same magnitude
% but roots in reciprocal conjugate positions.
% the polynomial is  $z^{-(\text{order of Ba})}*\text{conj}(Ba)(z^{-1})$ 
Ab = conj(fliplr(Ba));
Bb = Ba;

% We do not need to set the gain in the frequency to be = 1, we know
it
% will be 1 by construction

```

```

% % to check the zplane and the behaviour of the filter in frequency:
% figure;
% zplane(Bb, Ab);
% title('Z-plane of filter Hb');
% grid;
%
% [Hb, omega] = freqz(Bb, Ab, N, 'whole');
% figure,
% plot(omega./(2*pi), abs(Hb));
% title('DTFT of filter Hb');
% grid;

%% Hc

% can be defined as the polynomial with the same magnitude of Ba
% but roots in reciprocal conjugate positions.
Bc = conj(fliplr(Ba));
Ac = 1;

% % to check the zplane and the behaviour of the filter in frequency:
% figure;
% zplane(Bc, Ac);
% title('Z-plane of filter Hc');
% grid;
%
% [Hc, omega] = freqz(Bc, Ac, N, 'whole');
% figure,
% plot(omega./(2*pi), abs(Hc));
% title('DTFT of filter Hc');
% grid;

%% Hd

% we have to define a notch filter in correspondence of frequency f1.
% zeroes will be on the unit circle, so we need to modify their
absolute
% value to be 1.
% poles will be inside the unit circle, with the same phase of the
zeroes,
% and absolute value approaching 1.
zeroes_d = [exp(1i*2*pi*f1);exp(-1i*2*pi*f1)];
Bd = poly(zeroes_d);
poles_d = [0.95*exp(1i*2*pi*f1);0.95*exp(-1i*2*pi*f1)];
Ad = poly(poles_d);

% % to check the zplane and the behaviour of the filter in frequency:
% figure;
% zplane(Bd, Ad);
% title('Z-plane of filter Hd');
% grid;
%

```



```

% [Hd, omega] = freqz(Bd, Ad, N, 'whole');
% figure,
% plot(omega./(2*pi), abs(Hd));
% title('DTFT of filter Hd');
% grid;

%% 3

% [1 pt] Filter the signal w(n) with every filter defined in 2),
% obtaining wa(n), wb(n), wc(n), wd(n).

wa = filter(Ba, Aa, w);
wb = filter(Bb, Ab, w);
wc = filter(Bc, Ac, w);
wd = filter(Bd, Ad, w);

%% 4

% [2 pt] Compute the DFTs of the signals w(n), wa(n), wb(n), wc(n),
wd(n),
% and plot (using the function 'stem') their absolute values as a
function
% of the normalized frequency axis, starting from frequency 0.

% What do you expect to see in the DFTs of wa(n), wb(n), wc(n), wd(n),
% from the point of view of the DFTs amplitude with respect to the
original
% signal w(n)? Comment the results for each different signal.

% What do you expect to see in the DFTs of wa(n) and wc(n), from the
% point of view of the DFTs amplitude? And in the phase?

W = fft(w);
Wa = fft(wa);
Wb = fft(wb);
Wc = fft(wc);
Wd = fft(wd);

freq_axis = 0:1/N:1 - 1/N;

figure;
stem(freq_axis, abs(W));
title('Absolute value of the DFT of the original signal w(n)');
grid;
% we expect to see 6 peaks related to the three sinusoidal signals,
with
% the same amplitude

figure;
stem(freq_axis, abs(Wa));
title('Absolute value of the DFT of the signal wa(n)');
grid;

```

```
% The gain of the peaks has been modified by the filter  $H_a(z)$ , which
% attenuates the frequency  $f_1$ , but actually modifies the gain also in
the
% other frequencies. The 6 peaks will be shaped following the filter
 $H_a(f)$ .
```

```
figure;
stem(freq_axis, abs(Wb));
title('Absolute value of the DFT of the signal  $w_b(n)$ ');
grid;
% The filter is an all-pass filter, so we expect to see the 6 peaks
with
% (approximatively) the same amplitude of the original peaks
```

```
figure;
stem(freq_axis, abs(Wc));
title('Absolute value of the DFT of the signal  $w_c(n)$ ');
grid;
% The gain of the peaks has been modified by the filter  $H_c(z)$ , which
% attenuates the frequency  $f_1$ , but actually modifies the gain also in
the
% other frequencies. The 6 peaks will be shaped following the filter
 $H_c(f)$ .
```

```
figure;
stem(freq_axis, abs(Wd));
title('Absolute value of the DFT of the signal  $w_d(n)$ ');
grid;
% the filter is a notch filter, so we have attenuated the peaks
related to
% frequency  $f_1$ , leaving almost unaltered the other 4 peaks.
```

```
% The DFT amplitudes of  $w_a(n)$  and  $w_c(n)$  are the same, by construction.
% The phase of  $W_a(f)$ , being  $H_a(z)$  maximum phase, will include jumps of
 $2\pi$ ,
% while the phase of  $W_c(f)$ , being  $H_c(z)$  minimum phase, will not
include
%  $2\pi$ -jumps.
```

```
% to check the phase behaviour
% figure;
% stem(freq_axis, angle(Wa));
% title('Phase of the DFT of the signal  $w_a(n)$ ');
% grid;
% figure;
% stem(freq_axis, angle(Wc));
% title('Phase of the DFT of the signal  $w_c(n)$ ');
% grid;
```

