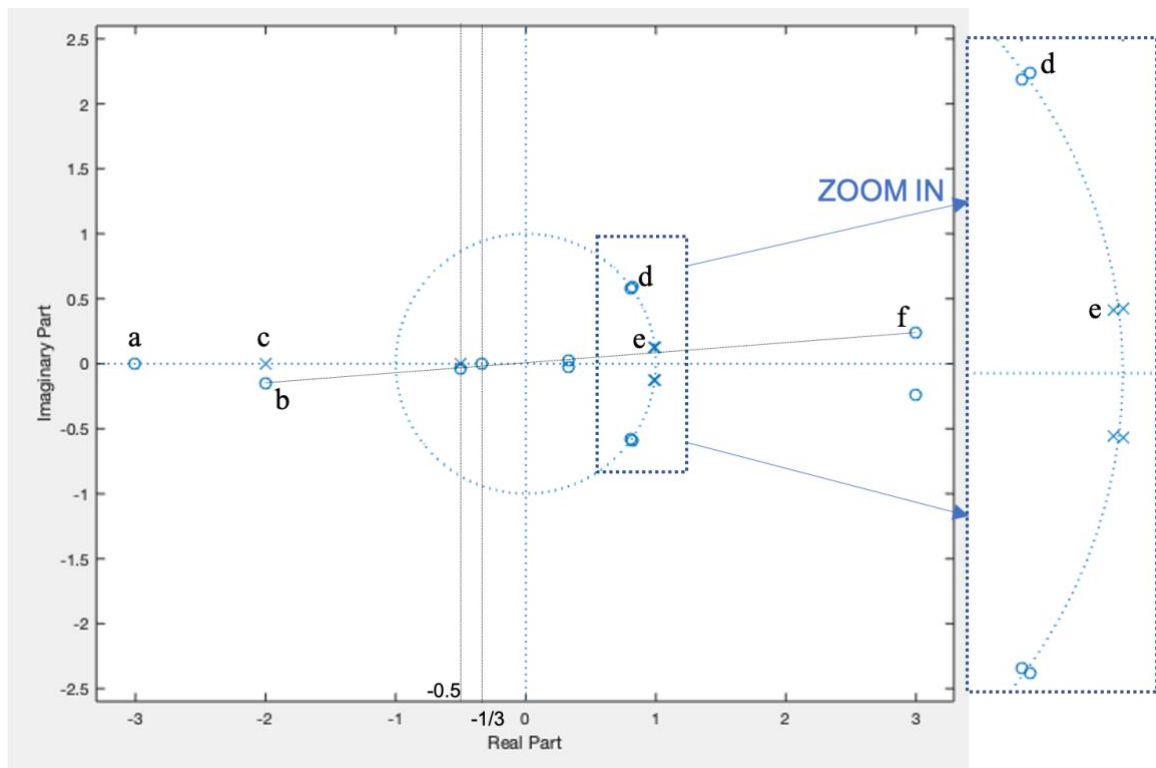# MATLAB part [12 pts]

February 18th, 2021

Text:

1. [2 pt] You are given three cosinusoidal signals: x, y and z. In particular, x has frequency = 1KHz, y has frequency = 1.6KHz and z has frequency = 8KHz. One entire cycle of the signal z is completed every 10 signal samples. The signal w is the summation of the three sinusoids, and the signals' duration is equal to one period of w. Define the signals x, y, z, w.

2. [4 pt] You are given the following zeros-poles diagram of a magnitude square function M(z).
   a = -3; b = -2exp(-2pi*j*0.0125); c = -2; d = 1.01exp(2pi*j*0.1); e = 0.99exp(2pi*j*0.02);
   f = 3exp(2pi*j*0.0125).

- Choose the poles and the zeroes to build a minimum phase filter H_min(z) with real coefficients such that, when filtering the signal w, only the frequency components related to signals z and x are attenuated. Keep all the minimum phase poles.
- Find the numerator and denominator of the filter H_min(z) such that the filter gain evaluated in frequency = 0 is equal to 0.5.

- Verify that H_min(z) has all the poles and zeros with minimum phase (use the function "zplane").
- Compute the frequency response of H_min(z) with the function "freqz", using the same number of samples of the w signal and using the flag "whole" to see the entire frequency spectrum. Plot the absolute value as a function of the frequency axis [Hz] starting from frequency = 0 [Hz]. Instead of the function "plot", it may be better to use "semilogy" to see the spectrum in logarithmic scale. Which signal component is strongly attenuated by the filter? Which is only slightly attenuated? Is there a component which is enhanced by the filter? Motivate your answers.

3. [3 pt] H_min(z) is part of the all-pass minimum-phase decomposition of a filter H(z), such that H(z) = H_min(z)*H_ap(z). In particular, H(z) has some zeros with absolute value = 3 and frequency equal to the lowest frequency of the sinusoidal components of w.
   - Find the all-pass component of the filter (setting the gain of H_ap(z) = 1).
   - Find the filter H(z).
   - Compute the frequency response of the filters H_ap(z) and H(z) with the function "freqz", using the same number of samples of the w signal and using the flag "whole" to see the entire frequency spectrum. Plot their absolute values as a function of the frequency [Hz]. (better to use again "semilogy").
   - Is there any difference between the absolute value of the frequency responses of H(z) and H_min(z)?

4. [2 pt] Filter the signal w with the filters H_min(z), H_ap(z) and H(z), defining the signals w_min, w_ap, w_h.
   - Plot the absolute value of the DFTs of the filtered signals as a function of frequency [Hz], starting from frequency = 0 Hz.
   - Which are the differences between the DFT of w_h and w_min?
   - Which are the differences between the DFT of w_h and w_ap?

5. [1 pt] Compute an approximation of the first 15 samples of the impulse responses of H_min(z) and H(z) (hint: you can use the function "filter").
   - Plot the two impulse responses with the function "stem". Which is the main characteristic of h_min(n) with respect to h(n)?

Solution:

```matlab
close all
clearvars
clc

%% 1.

% [2 pt] You are given three cosinusoidal signals: x, y and z.
% In particular, x has frequency = 1KHz, y has frequency = 1.6KHz and z
% has frequency = 8KHz. One entire cycle of the signal z is completed
% every 10 signal samples. The signal w is the summation of the three
% sinusoids, and the signals? duration is equal to one period of w.
% Define the signals x, y, z, w.

Fx = 1e3;
Fy = 1.6e3;
Fz = 8e3;

% One entire cycle of the signal z is completed every 10 signal samples
Pz_samples = 10;
Pz_seconds = 1/Fz;
Fs = Pz_samples / Pz_seconds;

% w is the summation of the three sinusoids
% the signals' duration is equal to one period of w.
% Px = Fs/Fx = 80 samples, Py = Fs/Fy = 50 samples, Pz = 10 samples --> Pw
= least common
% multiple(80, 50, 10) = 400 samples
Pw_samples = 400;
Pw_seconds = Pw_samples/Fs;
time = 0:1/Fs:Pw_seconds - 1/Fs;

x = cos(2*pi*Fx*time);
y = cos(2*pi*Fy*time);
z = cos(2*pi*Fz*time);
w = x + y + z;

%% 2.

% [4 pt] You are given the following zeros-poles diagram of a magnitude
square function M(z).
% a = -3; b = -2exp(-2pi*j*0.0125); c = -2; d = 1.01exp(2pi*j*0.1); e =
0.99exp(2pi*j*0.02);
% f = 3exp(2pi*j*0.0125).
% Choose the poles and the zeroes to build a minimum phase filter H_min(z)
% with real coefficients such that, when filtering the signal w,
% only the frequency components related to signals z and x are attenuated.
```

```matlab
% Keep all the minimum phase poles.
% Find the numerator and denominator of the filter H_min(z) such that
% the filter gain evaluated in frequency = 0 is equal to 0.5.
% Verify that H_min has all the poles and zeros with minimum phase
% (use the function "zplane").
% Compute the frequency response of H_min(z) with the function "freqz",
% using the same number of samples of the w signal and using the flag
"whole"
% to see the entire frequency spectrum. Plot the absolute value as a
function
% of the frequency axis [Hz] starting from frequency = 0 [Hz].
% Instead of the function "plot", it may be better to use "semilogy"
% to see the spectrum in logarithmic scale.
% Which signal component is strongly attenuated by the filter?
% Which is only slightly attenuated?
% Is there a component which is enhanced by the filter? Motivate your
answers.

% compute the normalized frequencies of the three sinusoids
Fxn = Fx/Fs; % 0.0125
Fyn = Fy/Fs; % 0.02
Fzn = Fz/Fs; % 0.1

% keep only the zeros which are inside the unit circle such that the
% resulting coefficients are real and Fzn and Fxn are attenuated.
zeroes = [1/1.01*exp(2*pi*1i*0.1), 1/1.01*exp(-2*pi*1i*0.1),
1/3*exp(2*pi*1i*0.0125), 1/3*exp(-2*pi*1i*0.0125)];
poles = [-1/2, 0.99*exp(2*pi*1i*0.02), 0.99*exp(-2*pi*1i*0.02)];

b_min = poly(zeroes);
a_min = poly(poles);

% find the numerator and denominator of the filter H_min such that
% the filter gain in frequency = 0 is equal to 0.5.
% In order to have gain = 0.5 in f = 0, we should impose that:
% k*B_min(z=1)/A_min(z=1) = 0.5 --> k = 0.5*sum(a_min)/sum(b_min).
k = 0.5*sum(a_min) / sum(b_min);
% multiply the numerator by k
b_min = k * b_min;

% Verify that H_min has all the poles and zeros with minimum phase
% (use the function "zplane").
figure;
zplane(b_min, a_min);

% Compute the frequency response of H_min(z) with the function "freqz",
% using the same number of samples of the w signal and using the flag
"whole"
```

```matlab
% to see the entire frequency spectrum. Plot the absolute value as a
function
% of the frequency axis [Hz] starting from frequency = 0 [Hz].
% Instead of the function "plot", it may be better to use "semilogy"
% to see the spectrum in logarithmic scale.
% Which signal component is strongly attenuated by the filter?
% Which is only slightly attenuated?
% Is there a component which is enhanced by the filter? Motivate your
answers.

N = length(w);
H_min = freqz(b_min, a_min, N, 'whole');
figure;
semilogy(0:Fs*1/N:Fs*(1-1/N), abs(H_min));
% The component which is strongly attenuated is Fz = 8KHz (the zeros
% related to this frequency have absolute value near 1).
% The component which is only slightly attenuated is Fx = 1KHz (the zeros
% related to this frequency have absolute value only = 1/3).
% The component which is enhanced is Fy = 1.6KHz (the poles
% related to this frequency have absolute value near 1).

%% 3.

% [3 pt] H_min(z) is part of the all-pass minimum-phase decomposition of a
% filter H(z), such that H(z) = H_min(z)*H_ap(z).
% In particular, H(z) has some zeros with absolute value = 3 and
% frequency equal to the lowest frequency of the sinusoidal components of
w.
% Find the all-pass component of the filter (setting the gain of H_ap(z) =
1).
% Find the filter H(z).
% Compute the frequency response of the filters H_ap(z) and H(z)
% with the function "freqz", using the same number of samples of the w
signal
% and using the flag "whole" to see the entire frequency spectrum.
% Plot their absolute values as a function of the frequency [Hz].
% (better to use again "semilogy").
% Is there any difference between the absolute value of the
% frequency responses of H(z) and H_min(z)?

% H(z) has some zeros with absolute value = 3 and
% frequency equal to the lowest frequency of the sinusoidal components of
w
% lowest frequency = Fxn
zeroes_h = [3*exp(1i*2*pi*Fxn), 3*exp(-1i*2*pi*Fxn)];

% The all-pass component of the filter contains the zeros outside the unit
% circle and the poles which are conjugate reciprocal of these zeroes.
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```matlab
poles_ap = 1./conj(zeroes_h);
zeroes_ap = zeroes_h;

b_ap = poly(zeroes_ap);
a_ap = poly(poles_ap);

% Set the filter gain = 1
k = sum(a_ap) / sum(b_ap);
% multiply the numerator b_ap by k
b_ap = k*b_ap;

% The filter H(z) = H_ap(z) * H_min(z) --> exploit the convolution
property
b = conv(b_ap, b_min);
a = conv(a_ap, a_min);

% Compute the frequency response of the filters H_ap(z) and H(z)
% with the function "freqz", using the same number of samples of the w
signal
% and using the flag "whole" to see the entire frequency spectrum.
% Plot their absolute values as a function of the frequency [Hz].
% (better to use again "semilogy").
% Is there any difference between the absolute value of the
% frequency responses of H(z) and H_min(z)?

H_ap = freqz(b_ap, a_ap, N, 'whole');
H = freqz(b, a, N, 'whole');

figure;
semilogy(0:Fs*1/N:Fs*(1-1/N), abs(H_ap));
leg{1} = 'All-pass filter';
hold on;
semilogy(0:Fs*1/N:Fs*(1-1/N), abs(H));
leg{2} = 'Filter H(z)';
hold on;
semilogy(0:Fs*1/N:Fs*(1-1/N), abs(H_min), '--');
leg{3} = 'Minimum phase filter';
legend(leg);
% The magnitude response of H(z) and H_min(z) is the same, because
% |H_ap(z)| is flat and equal to 1 for all frequencies.

%% 4.

% [2] Filter the signal w with the filters H_min(z), H_ap(z) and H(z),
% defining the signals w_min, w_ap, w_h.
% Plot the absolute value of the DFTs of the filtered signals
% as a function of frequency [Hz], starting from frequency = 0 Hz.
% Which are the differences between the DFT of w_h and
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```matlab
% w_min? Which are the differences between the DFT of w_h and w_ap?

w_min = filter(b_min, a_min, w);
w_ap = filter(b_ap, a_ap, w);
w_h = filter(b, a, w);

W_min = fft(w_min);
W_ap = fft(w_ap);
W_h = fft(w_h);

freq_axis = 0:Fs*1/length(w):Fs*(1-1/length(w));

figure;
plot(freq_axis, abs(W_min));
leg{1} = 'Minimum-phase filtered signal';
hold on;
plot(freq_axis, abs(W_ap), '*-');
leg{2} = 'All-pass filtered signal';
plot(freq_axis, abs(W_h), 'o-');
leg{3} = 'Signal filtered with H(z)';
legend(leg);

% There are no differences between the |DFT| of w_h and w_min, because, as
% previously shown, |H_min(f)| = |H(f)|. The |DFT| of w_ap contains also
the
% peak in 8KHz, which was basically deleted by the filters H(z) and
% H_min(z).

%% 5.

% [1] Compute an approximation of the first 15 samples of the impulse
responses of H_min(z) and H(z)
% (hint: you can use the function "filter").
% Plot the two impulse responses with the function "stem". Which is the
% main characteristic of h_min(n) with respect to h(n)?

N = 15;
delta = zeros(1, N);
delta(1) = 1;
h_min = filter(b_min, a_min, delta);
h = filter(b, a, delta);

figure,
stem(h_min);
leg{1} = 'Minimum-phase filter';
hold on;
stem(h);
```

```matlab
leg{1} = 'Filter h(n)';

% Minimum phase filters have their energy concentrated in the first
% samples. Notice that the filter h(n) has its energy more spread in all
% the samples while h_min(n) has the maximum peak in n = 0.
```