MultimediaSignal Processing 1st Module

MATLAB part [11 pts]

September 3rd, 2020

Text:

1. [2.5 pt] x and y are sinusoidal signals. Their normalized frequencies are, respectively, 0.04 and 0.05. Their amplitude is 1, and the sampling period is 0.02 seconds.
    - Define the signal z = x + y with duration 1 second and plot it as a function of its samples.
    - Which is the period of the signal z?
2. [2.5 pt] Compute the DFT of the signal z.
    - As z is the sum of two sinusoids, what do you expect from its DFT?
    - Plot the DFT magnitude as a function of normalized frequencies starting from 0, using the function "stem".
    - Is this result equal to the theoretical spectrum of the sum of two sinusoids? Motivate your answer.
    - Which is the resolution of the z spectrum?
3. [6 pt] We want the z spectrum to match with the theoretical one. In order to do this, you can choose among these options:
    - (a) zero-pad the signal z;
    - (b) interpolate the signal z;
    - (c) evaluate the signal z over a longer time window;

   Investigate the three cases, specifying which is the right answer and motivating it, commenting on the results of each case. For all the three cases, compute the DFT of the obtained signal and plot it versus normalized frequencies using the function "stem". Be careful in correctly defining the normalized frequency axis!
    - Regarding (a), you choose the amount of zero-padding.
    - Regarding (b), you choose the interpolation factor and use a filter with 65 taps.
    - Regarding (c), which are the possible values of the longer time window? Which is the minimum one? Select the minimum possible value.

Solution:

```
%% 1 [2.5 pt]

% x and y are sinusoidal signals.
% Their normalized frequencies are, respectively, 0.04 and 0.05.
% Their amplitude is 1, and the sampling period is 0.02 seconds.
% Define the signal z = x + y with duration 1 second, and plot it as a
function of its samples.
% Which is the period of the signal z?

fx = 0.04;
fy = 0.05;
Ts = 0.02;
duration = 1;
% number of signal samples
N = 1 / Ts;
n_axis = 0:N-1;

z = cos(2*pi*fx*n_axis) + cos(2*pi*fy*n_axis);

figure;
plot(z);

% in order to compute the period of z:
% Tx = 1/0.04 = 25; Ty = 1/0.05 = 20;
% the period of z is the least common multiple between them
Tz = 100;

%% 2 [2.5 pt]

% Compute the DFT of the signal z. As z is the sum of two sinusoids, what
do you expect from its DFT?
% Plot the DFT magnitude as a function of normalized frequencies starting
from
% 0, using the function "stem".
% Is this result equal to the theoretical
% spectrum of the sum of two sinusoids? Motivate your answer.
% which is the resolution of the z spectrum?

Z_f = fft(z);
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norm_f_axis = 0: 1/N: 1 - 1/N;
figure;
stem(norm_f_axis, abs(Z_f));

% The spectrum of the sum of two sinusoids is zero everywhere, except for
4 peaks in normalized
% frequencies f1, f2, 1-f1, 1-f2.
% The spectrum of z does not have this behaviour because N is not a
% multiple of the signal period, therefore the DFT contains spurious
% content.

% the z spectrum resolution depends on the number of signal samples, N.
resolution = 1 / N;

%% 3 [6 pt]

% We want the z spectrum to match with the theoretical one.
% In order to do this, you can choose among these options:
% (a) zero-pad the signal z;
% (b) interpolate the signal z;
% (c) evaluate the signal z over a longer time window;
% Investigate the three cases, specifying which is the right answer and
motivating it, commenting on the results of each case.
% For all the three cases, compute the DFT of the obtained signal and plot
it versus
% normalized frequencies using the function "stem". Be careful in
correctly defining the normalized
% frequency axis!
% Regarding (a), you choose the amount of zero-padding.
% Regarding (b), you choose the interpolation factor and use a filter with
65
% taps. Regarding (c), which are the possible values of the longer time
window? Which is the minimum one?
% Select the minimum possible value of the longer time window.

%% (a) zero-pad the signal z

% for example, try adding N_pad = 50 samples, in order to achieve a total
amount
% of 100 samples, so that the spectrum resolution is enough to see the two
% distinct sinusoids.

N_pad = 50;
z_pad = padarray(z, [0, N_pad], 0, 'post');

Z_f_pad = fft(z_pad);
norm_f_axis_pad = 0: 1/length(z_pad): 1 - 1/length(z_pad);
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```
figure;
stem(norm_f_axis_pad, abs(Z_f_pad));

% zero-padding the signal does not help. You are not introducing new
% information, but just interpolating the spectrum of z.

%% (b) interpolate the signal z.

% use L = 2, in order to obtain a signal with 100 samples, so that the
spectrum resolution is enough to see the two
% distinct sinusoids.

L = 2;
z_up = zeros(1, length(z) * L);
z_up(1:L:end) = z;

lpf = fir1(64, 1/L);
z_int = L*filter(lpf, 1, z_up);

Z_f_int = fft(z_int);
norm_f_axis_int = 0: 1/length(z_int): 1 - 1/length(z_int);

figure;
stem(norm_f_axis_int, abs(Z_f_int));

% interpolating the signal does not help. You are not introducing new
% information on the signal z, but just interpolating the already known
samples.

%% (c) evaluate the signal z over a longer time window

% the possible values of the longer time window are all the multiples of
% the period of z. Therefore, N_longer = 100, 200, 300, etc...
% the minimum possible value is the z period = 100.

N_longer = 100;
n_longer_axis = 0:N_longer-1;
% define the new signal, evaluated over a longer time window.
z_longer = cos(2*pi*fx*n_longer_axis) + cos(2*pi*fy*n_longer_axis);

Z_f_longer = fft(z_longer);
norm_f_axis_longer = 0: 1/length(z_longer): 1 - 1/length(z_longer);

figure;
stem(norm_f_axis_longer, abs(Z_f_longer));

% Evaluating the signal z over multiples of its period is the solution.
% this way, there is no spurious content in the DFT and the solution
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% matches with the theoretical one.
```