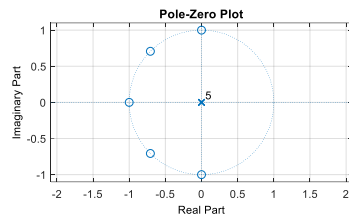


Ex.1 (Pt.15)

A signal is sampled at 10kHz and we want to downsample it in real-time at 5kHz, in order to do this we can use the following low pass filter $H_1(z)$ whose pole-zero plot is the following:



[Pts. 1] Is it a FIR or an IIR Filter?

[Pts. 7] Provide its z transform [preserving the magnitude of the continuous component equal to 1].

[Pts. 3] We want to use the Overlap-and-Save technique in order to filter the signal working in the frequency domain; we will work on blocks of 10 samples. Provide only the formula in order to generate the elements of the DFT Matrix.

[Pts. 3] Detail the procedure (without calculations), starting from blocks of 10 samples, in order to filter the signal using the DFT and to apply the Overlap And Save procedure specifying the number of samples obtained at every step and the number of discarded ones.

[Pts. 1] Describe (without calculations) the last step in order to downsample the signal.

Ex.2 (Pt.7)

A filter is described by the following equation $y(n) = x(n) + x(n-2) + \frac{4}{9}y(n-2)$.

[Pts. 3] Provide its pole-zero plot.

[Pts. 4] When applied to a signal sampled at 40kHz, provide the Gain/Attenuation at these frequencies: 0Hz, 10kHz, 20kHz.

Ex.3 (Pt. 11 – MATLAB code)

1. [2.5 pt] A filter $H(z)$ is defined by:

- a denominator $D(z) = (1 + 0.5 \exp(i\frac{\pi}{4})z^{-1})(1 + 0.5 \exp(-i\frac{\pi}{4})z^{-1})(1 + 0.9z^{-1})$
- a numerator $N(z) = (1 + 1.1 \exp(i\frac{\pi}{3})z^{-1})(1 + 1.1 \exp(-i\frac{\pi}{3})z^{-1})(1 - 0.8z^{-1})$
- Find the polynomials $D(z)$ and $N(z)$ exploiting the convolution property
- Plot zeroes and poles in the Z plane

2. [5 pt] Write the code (using MATLAB syntax) of a function 'allpass_minphase.m', which receives as inputs the numerator and the denominator of a filter and returns its allpass-minimum phase decomposition, in the form of $N_{ap}(z)$, $D_{ap}(z)$, $N_{min}(z)$, $D_{min}(z)$.

3. [1 pt] Test the function on the filter $H(z)$.

- Which is the relationship between zeroes and poles in $H_{ap}(z)$?
- Which is the common feature of the roots in $H_{min}(z)$?

4. [2.5 pt] Define the sinusoidal signal x , sampled at $F_s = 4.8\text{kHz}$, with amplitude 0.8, frequency 800 Hz, duration 2 seconds

- Filter the signal x with the filter $H_{ap}(z)$ defined above.
- Plot the magnitude of the DFT of the filtered signal as a function of normalized angular frequencies defined between $[0, 2\pi)$.
- What do you expect to see in the DFT of the initial signal x and in DFT of the filtered signal?

Solutions

Ex.1

It is a FIR filter, we can obtain its z transform composing the conjugate zeros:

$$\begin{aligned} H_1(z) &= A(1+z^{-1}) \cdot \left(1 - 2\cos\left(\frac{3}{4}\pi\right)z^{-1} + z^{-2}\right) \cdot \left(1 - 2\cos\left(\frac{\pi}{2}\right)z^{-1} + z^{-2}\right) = \\ &= A(1+z^{-1}) \cdot (1+\sqrt{2}z^{-1} + z^{-2}) \cdot (1+z^{-2}) = (1+z^{-1} + z^{-2} + z^{-3}) \cdot (1+\sqrt{2}z^{-1} + z^{-2}) = \\ &= A\left[1 + (1+\sqrt{2})z^{-1} + (2+\sqrt{2})z^{-2} + (2+\sqrt{2})z^{-3} + (1+\sqrt{2})z^{-4} + z^{-5}\right] \end{aligned}$$

Preserving the continuous component equal to 1 mean that the magnitude of the filter when $z = 1$ is equal to 1:

$$H_1(z=1) = A(1+1+\sqrt{2}+2+\sqrt{2}+2+\sqrt{2}+1+\sqrt{2}+1) = 1 \Rightarrow A = \frac{1}{8+4\sqrt{2}}$$

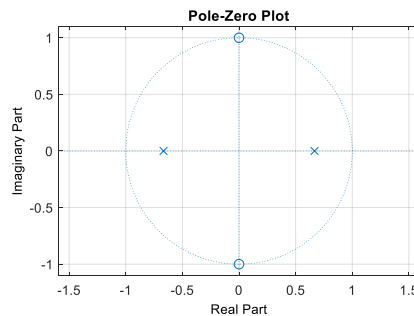
The elements of the DFT Matrix will be $W_{r,c} = e^{-j\frac{2\pi}{10} \cdot r \cdot c}$

For the Overlap and Save procedure see the details on the lectures' slides. The Overlap and Save, due to the circular convolution will produce an output of 10 samples where the first 5 are affected by the overlap of the tail with the head (due to the circular convolution), so we can keep only the last five samples removing the first five from the output. The next block of 10 samples will start from the 6th sample of the previous block.

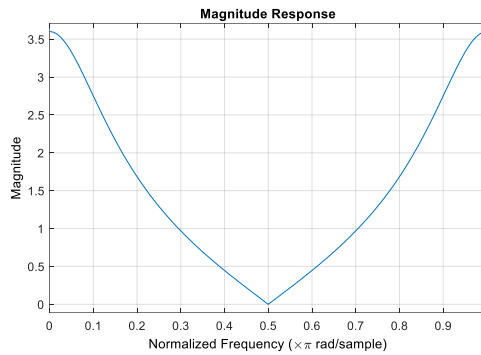
The last step of the downsampling consists in removing one sample every two samples from the output of the filter.

Ex.2

The z transform is $H(z) = \frac{1+z^{-2}}{1-\frac{4}{9}z^{-2}}$; the pole zero plot is:



The considered frequencies in normalized units are: $0, \frac{\pi}{2}, \pi$ and the Gain/Attenuation is the following:



And

$$H(z=1) = H(z=-1) = \frac{18}{5}$$

$$H(z = \pm j) = 0$$

Ex.3

%% 1.

% Find the polynomials D(z) and N(z) exploiting the convolution property

```
D_1 = [1, 0.5 * exp(1i*pi/4)];
D_2 = [1, 0.5 * exp(-1i*pi/4)];
D_3 = [1, 0.9];
```

```
N_1 = [1, 1.1 * exp(1i*pi/3)];
N_2 = [1, 1.1 * exp(-1i*pi/3)];
N_3 = [1, -0.8];
```

```
D = conv(conv(D_1, D_2), D_3);
N = conv(conv(N_1, N_2), N_3);
```

% Plot zeros and poles in the Z plane

```
figure;
zplane(N, D);
```

%% 2.

```
% Write the code (using MATLAB syntax) of a function ?allpass_minphase.m?,
% which receives as inputs the numerator and the denominator of a filter
% and returns its allpass-minimum phase decomposition,
% in the form of Nap(z), Dap(z), Nmin(z), Dmin(z).
% NB: functions should be defined in different files or at the end of the
% script --> check the end of this script
```

%% 3.

% Test the function on the filter H(z)

```
[N_ap, D_ap, N_min, D_min] = allpass_minphase(N, D);
```

```
% Which is the relationship between zeroes and poles in Hap(z)?
% --> poles and zeroes occurs in conjugate reciprocal positions
% Which is the common feature of the roots in Hmin(z)?
% --> all poles and zeros are inside the unit circle
```

%% 4.

```

% Define the sinusoidal signal x, sampled at Fs = 4.8KHz, with amplitude
%0.8,
% frequency 800 Hz, duration 2 seconds

ampl = 0.8;
f0 = 800;
Fs = 4.8e3;
duration = 2;
time = 0:1/Fs:duration;

x = ampl * cos(2*pi*f0*time);

% Filter the signal x with the filter Hap(z) defined above

y = filter(N_ap, D_ap, x);

% Plot the magnitude of the DFT of the filtered signal as a function of
% normalized angular frequencies defined between [0, 2?).

Yf = fft(y);
N_samples_fft = length(y);
norm_ang_freq_axis = 0: 2*pi/N_samples_fft:2*pi*(N_samples_fft- 1)/
N_samples_fft;

figure;
plot(norm_ang_freq_axis, abs(Yf));

% What do you expect to see in the DFT of the initial signal x
% and in DFT of the filtered signal?

% the input signal has 2 peaks corresponding to normalized angular
% frequencies pi/3 and - pi/3.
% The all-pass filter has constant gain in the frequency spectrum -->
% we expect to see the same spectrum of the input signal, multiplied by %the
% gain of the filter.

%% function code

function [N_ap, D_ap, N_min, D_min] = allpass_minphase(N, D)

% compute the zeroes and the poles
zeroes = roots(N);
poles = roots(D);

% The minimum phase system contains zeroes and poles inside the unit circle
% plus the conjugate reciprocal of zeros outside the unit circle
z_min = zeroes(abs(zeroes) <= 1);
p_min = poles(abs(poles) <=1);

% check if there are zeroes outside the unit circle
if any(abs(zeroes) > 1)
z_min = [z_min; 1./conj(zeroes(abs(zeroes) > 1))];
% The allpass decomposition contains the zeroes outside the unit circle %and
% poles which are the conjugate reciprocal of zeroes outside the unit %circle
p_ap = 1./conj(zeroes(abs(zeroes) > 1));
z_ap = zeroes(abs(zeroes) > 1);
end

% define the polynomials of H_min(z)
N_min = poly(z_min);
D_min = poly(p_min);
% define the polynomials of H_ap(z)
N_ap = poly(z_ap);
D_ap = poly(p_ap);

```

end