date: 16/01/2019

Ex.1 (Pt.12)

A digital signal is made of 3 samples: $x[n]=\{2,3,-1\}$. Working **exclusively** in the Discrete Frequencies domain, we want to find its continuous component (0 frequency) removing the high frequencies.

- [3 Pts.] Define the $\mathbf{W}$ Matrix to perform the DFT of the 3 samples signal.
- [2 Pts.] Calculate $X[k]$, the DFT of the input signal.
- [3 Pts.] Define the filter $H[k]$ in the frequency domain in order to remove the high frequencies and extract the filtered signal $Y[k]$.
- [4 Pts.]Transform $H[k]$ and $Y[k]$ into their time domain representation $h[n]$ and $y[n]$ using the iDFT.

Ex.2 (Pt.11)

A maximum phase filter has the following transfer function:

$$H_M(z)=\frac{\left(1-2\sqrt{2}z^{-1}+4z^{-2}\right)\left(1+2\sqrt{2}z^{-1}+4z^{-2}\right)}{\left(4+z^{-2}\right)}$$

1. [2 Pts.] Provide its zeros-poles plot
2. [4 Pts.] Provide its minimum phase version, $H_m(z)$, with **exactly** the same magnitude response.
3. [5 Pts.] A signal $x(t)=\cos(2\pi 1000t)$ is sampled at 4kHz. Define the outputs, $y_M[n]$ and $y_m[n]$ with the proper **amplitude** and **phase** from the two filters, $H_M(z)$ and $H_m(z)$.

Ex.3 (Pt. 11 – MATLAB code)

Given the signal x from the wavefile 'TomsDiner.wav', we want to filter it with a generic low-pass filter with one zero and one pole, and plot the comparison in the time domain

1. [2pt.] Load the wave file into x
2. [3pt.] Define a low pass filter h with 1 zero and 1 pole, placing them accordingly
3. [2 pt.] compute the difference equation representation of the IIR filter
4. [2pt.] Compute y as the filtering of the signal x with h
5. [2pt] Plot x and y as two subplots of the same figure

Solutions

Ex.1

$$\mathbf{W}=\begin{bmatrix} e^{-j\frac{2\pi}{3}\cdot 0\cdot 0} & e^{-j\frac{2\pi}{3}\cdot 0\cdot 1} & e^{-j\frac{2\pi}{3}\cdot 0\cdot 2} \\ e^{-j\frac{2\pi}{3}\cdot 1\cdot 0} & e^{-j\frac{2\pi}{3}\cdot 1\cdot 1} & e^{-j\frac{2\pi}{3}\cdot 1\cdot 2} \\ e^{-j\frac{2\pi}{3}\cdot 2\cdot 0} & e^{-j\frac{2\pi}{3}\cdot 2\cdot 1} & e^{-j\frac{2\pi}{3}\cdot 2\cdot 2} \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$X[k]=\mathbf{W}\cdot\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}=\begin{bmatrix} 4 \\ 1-3.4641j \\ 1+3.4641j \end{bmatrix}$$

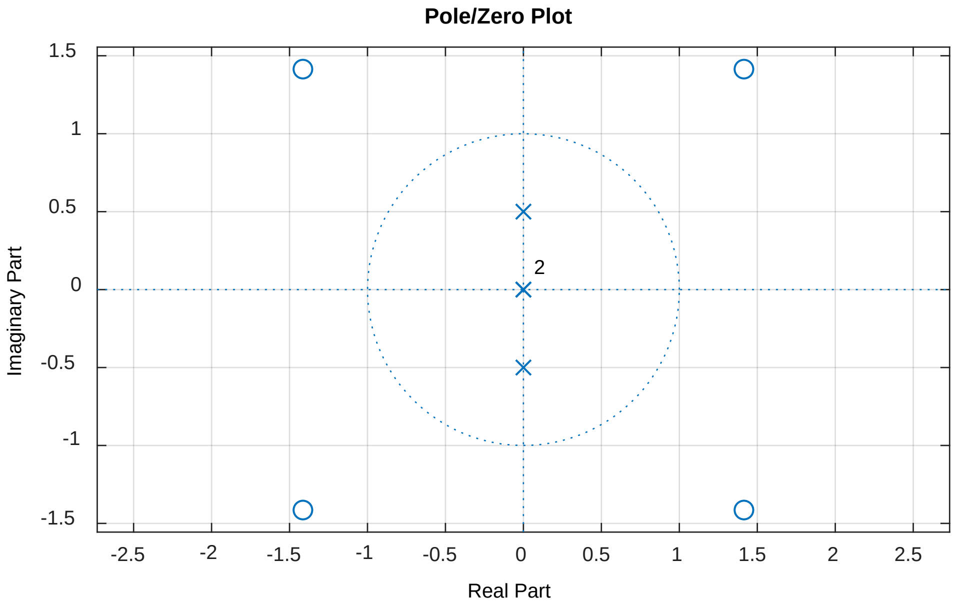
$$H[k]=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, Y[k]=H[k].*X[k]=\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$y[n]=iDFT\left(Y[k]\right)=\mathbf{W}^{-1}Y[k]=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 4/3 \\ 4/3 \\ 4/3 \end{bmatrix}$$

$$h[n]=iDFT\left(H[k]\right)=\mathbf{W}^{-1}H[k]=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Ex.2

The zeros poles plot is:



The minimum phase filter will have the zeros in the inverse conjugate position with respect to their actual position:

$$H_m(z) = 16\frac{\left(1-\frac{\sqrt{2}}{2}z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{\sqrt{2}}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}{\left(4+z^{-2}\right)}$$

The normalized frequency of the signal is $\omega = 2\pi\frac{1000}{4000} = \frac{\pi}{2}$ and the filters responses will be:

$$\left|H_M\left(z=e^{j\frac{\pi}{2}}=j\right)\right|=\frac{1+16}{4-1}=\frac{17}{3}$$

$$\left|H_m\left(z=e^{j\frac{\pi}{2}}=j\right)\right|=16\frac{1+\frac{1}{16}}{4-1}=\frac{17}{3}$$

$$\angle\left(H_M\left(z=e^{j\frac{\pi}{2}}=j\right)\right)=-2\pi$$

$$\angle\left(H_m\left(z=e^{j\frac{\pi}{2}}=j\right)\right)=0$$

$$y_M[n]=y_m[n]=\cos\left(\frac{\pi}{2}n\right)$$

Ex.3

```
%% 1.[2pt.] Load the wave file into x
[x,Fs]=audioread('TomsDiner.wav');
%% 2.[3pt.] define a low pass filter h with 1 zero and
1 pole, placing them accordingly
% Lowpass filter
% Zeros close to omega=pi → -1
% Poles close to omega=0 → 0.9 (inside the unit
circle!)
z=-1;
p=0.8;
%% 3. [2 pt.] compute the difference equation
representation of the IIR filter
b=poly(z);
a=poly(p);
%% 4.[2pt.] Compute y as the filtering of the signal x
with h
y=filter(b,a,x);
%% 5.[2pt] Plot x and y as two subplots of the same
figure
figure;
t_x=[0:length(x)-1]/Fs;
t_y=[0:length(y)-1]/Fs;
subplot(2,1,1);
plot(t_x,x);
title('Input of the filter x')
xlabel('Time [s]');
ylabel('x[n]');
subplot(2,1,2);
plot(t_y,y);
title('Output of the filter y')
```

```
xlabel('Time [s]');
ylabel('y[n]');
```