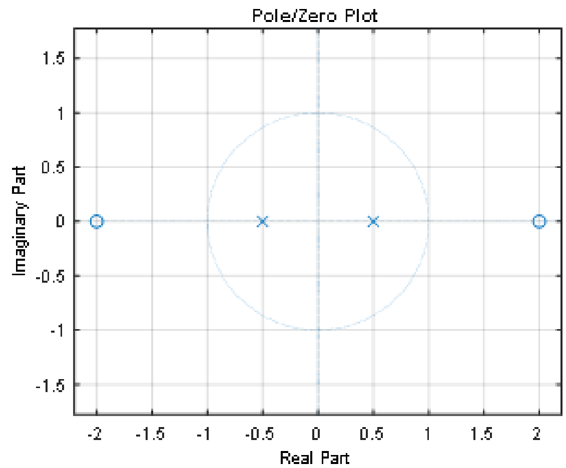
date: 25/6/2018

Ex.1 (Pt.11)

A digital filter has the following Pole/zero plot:



1. [3 pts] Define its z-transform
2. [2 pts] Describe the kind of filter and its behaviour
3. [6 pts] A continuous signal $x(t)=10\cos(2\pi 500t)+10\cos(2\pi 1000t)$ is sampled at 4 kHz and then filtered with the above filter. The output signal will be $y[n]=A_1\cos(k_1 n+\phi_1)+A_2\cos(k_2 n+\phi_2)$, provide the values of the output parameters $A_1, k_1, \phi_1, A_2, k_2, \phi_2$.

Ex.2 (Pt.11 <u>NOT to be done in MATLAB</u>)

Let x(n) be a discrete-time rectangular pulse of length $L=4$, i.e. $x(n)=u(n)-u(n-4)$,and $h(n)$ be a discrete-time ramp of length $M=3$, i.e. $h(n)=\{1,2,3\}$.

The output $y(n)$ is evaluated as an 6-points sequence obtained from the inverse DFT of the product $Y_N(k)=X_N(k)H_N(k)$.

1) Define how can be obtained the 36 coefficients of the **W** matrix to get the DFT of $x(n)$ and $h(n)$: <u>do not</u> list all coefficients but only the formula in order to get W_{rc} where *r* and *c* represent raw and column index of the matrix element.
2) Get the 6 values of $y(n)$ working in the time domain,
3) Will time-aliasing be present?

Ex.3 (Pt. 11 – MATLAB code)

Given the signal x[n]: $x[n] = \cos(2\pi f_1 n) + \sin(2\pi f_2 n)$

with $f_1 = 0.15, f_2 = 0.4$, n=0,1,…, 100

1. [3 pts] compute the signal y[n] as the filtering of

$$h[n] = [0.0350, 0.4650, 0.4650, 0.0350]$$

with n=[0,1,2,3] to the signal x[n] in the time domain using the conv function

2. [2 pts] plot y[n] in the time domain;
3. [4 pts] compute Y(k) as the filtering of h(n) to the signal x(n) in the DFT domain;
4. [2 pts] plot Y(k) (magnitude and phase) in the frequency domain

Solutions

Ex.1

The filter has the following z-transform:

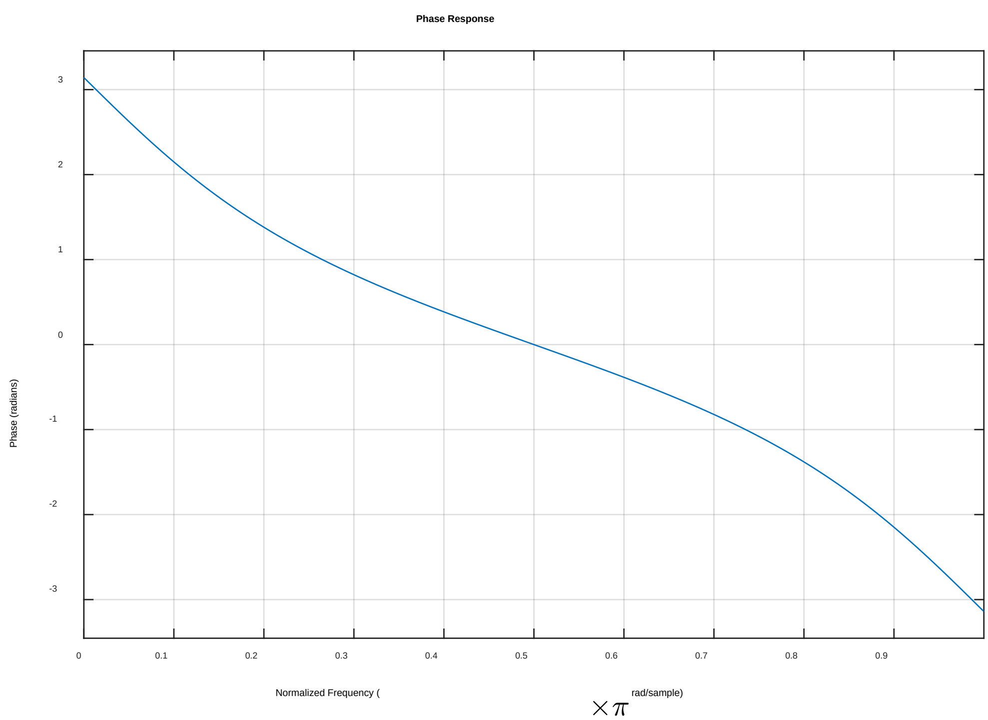
$$H(z) = A\frac{\left(1-2z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = A\frac{1-4z^{-2}}{1-\frac{1}{4}z^{-2}}$$

In order to set $\left|H(z)\right| = 1$ at the zero frequency (z=1) we can set

$$\left|H(z)\right|_{z=1} = 1 \Rightarrow A\left|\frac{1-4z^{-2}}{1-\frac{1}{4}z^{-2}}\right|_{z=1} = 1 \Rightarrow A = \frac{1}{4}$$

The filter is a stable all pass filter since for every pole inside the unit circle there is a zero in its inverse conjugate position.

The magnitude of the filter will always be 1 (0dB) while its phase will be the following:



The input signal will be:

$$x[n] = 10\cos\left(\frac{2\pi 500}{4000}n\right) + 10\cos\left(\frac{2\pi 1000}{4000}n\right) = 10\cos\left(\frac{\pi}{4}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$$

The two sinusoids will have the same frequency and amplitude at the output but will have a different phase:

$$\measuredangle H(z)\Big|_{z=e^{j\pi/4}} = \measuredangle\left(\frac{1}{4}\frac{1-4e^{-j\pi/2}}{1-\frac{1}{4}e^{-j\pi/2}}\right) = \measuredangle(1+4j) - \measuredangle\left(1+\frac{1}{4}j\right) = \tan^{-1}(4) - \tan^{-1}\left(\frac{1}{4}\right) = 1.0808$$

$$\measuredangle H(z)\Big|_{z=e^{j\pi/2}} = \measuredangle\left(\frac{1}{4}\frac{1-4e^{-j\pi}}{1-\frac{1}{4}e^{-j\pi}}\right) = \measuredangle(1+4) - \measuredangle\left(1+\frac{1}{4}\right) = 0$$

So the output signal will be

$$y[n] = 10\cos\left(\frac{\pi}{4}n + 1.0808\right) + 10\cos\left(\frac{\pi}{2}n\right)$$

Ex.2 (Pt.11 NOT to be done in MATLAB)

The linear convolution of $x(n)$ and $h(n)$ will have a length of 4+3-1=6, so, since N=6 there will be no time-domain aliasing.

The DFT coefficient will be $e^{-j\frac{2\pi}{6}r\cdot c}$

The output, convolving the x and h will be: $\{1,3,6,6,5,3\}$

Ex.3

```
% Given the signal x[n]=cos(2*pi*f1*n)+sin(2*pi*f2*n);
% with f1=0.15, f2=0.4, n=0,1,...,100
n=0:100;
f1=0.15; f2=0.4;
x=cos(2*pi*f1*n)+sin(2*pi*f2*n);

% 1. compute the signal y as the filtering of
% h=[0.0350, 0.4650, 0.4650, 0.0350];n=0,1,2,3
% to the signal x in the time domain;using the conv function

Ny=length(x)+length(h)-1;y=conv(x,h);

% 2. plot y in the time domain;
figure;plot([0:Ny-1],y);
xlabel('n');ylabel('y(n)');
title('Filtering in the time domain')

% 3. compute Y as the filtering of h to the signal x in the
%    DFT domain;

Y=fft(x,Ny).*fft(h,Ny);

% 4. plot Y (magnitude and phase) in the frequency domain
w=linspace(0,2*pi,Ny);
figure;
subplot(2,1,1);plot(w,abs(Y));
xlabel('\omega');ylabel('|Y(k)|')
```

```
title('Magnitude');
subplot(2,1,2);plot(w,angle(Y));
xlabel('\omega');ylabel('\angle Y(k)')
title('Phase');
```