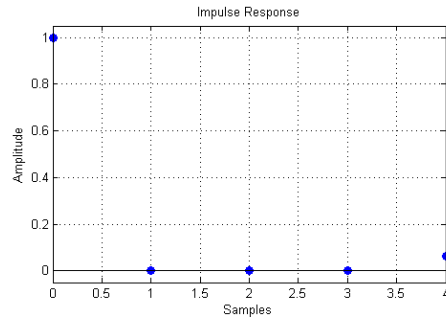


**Ex.1 (Pt.13)**

An audio signal is sampled at 8kHz, the transmission channel presents an echo and its impulse response is show below: (the values are [1,0,0,0,1/16])

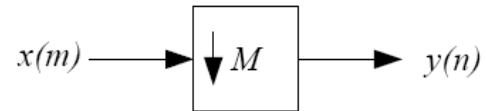
1. Define the z-transform of the transmission channel
2. Plot the zero poles diagram of the filter.
3. Find the exact position of zeros.
4. Find the exact gain/attenuation for a sinusoid at 1kHz
5. Find the exact gain/attenuation for a sinusoid at 2kHz
6. Define an IIR filter to completely remove the echo providing its difference equation.



**Ex.2 (Pt.9)**

Describe the downsampling of an order of 4 of a signal: (where M=4).

1. If the spectrum of original signal extends from  $-\pi/3$  to  $\pi/3$  (in normalized frequencies) draw the final spectrum after downsampling (quoting both axes and indicating central frequencies for all the replicas)
2. If aliasing is present in the downsampled signal, suggest a way to avoid it (keeping the same downsampling rate).



**Ex.3 (Pt. 11 – MATLAB code)**

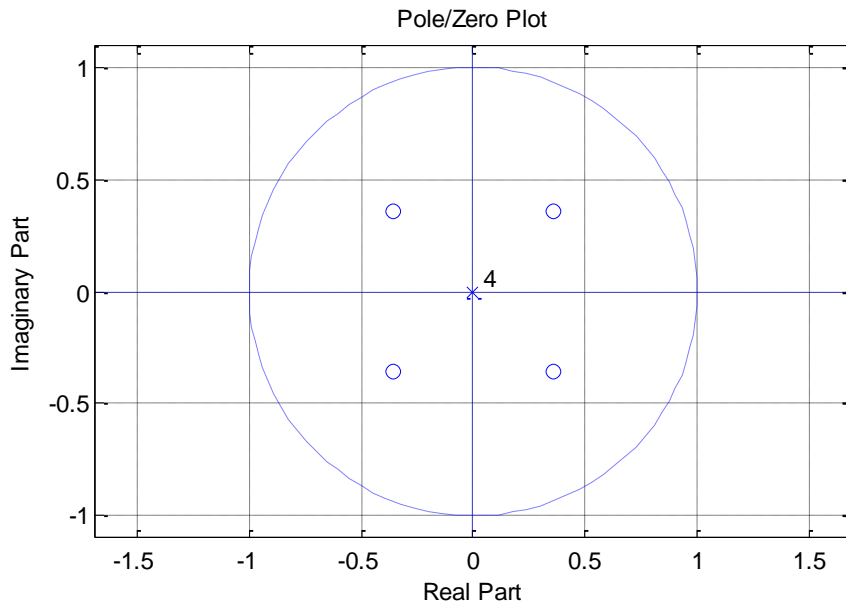
Given the filter  $h=[0,2,1]$  and the signal  $x=[4\ 3\ 2\ 6\ 4\ 3]$ ; with time indices starting both from zero

- 1) compute  $y$  as the linear convolution of  $h$  and  $x$  in the time domain (you can use MATLAB functions)
- 2) compute  $y_2$  as the linear convolution of  $h$  and  $x$  in the frequency domain (you can use MATLAB functions)
- 3) Plot the squared error between  $y$  and  $y_2$
- 4) Plot the two signals  $y$  and  $y_2$  in the same plot

## Solutions

### Ex.1

The z-transform will be:  $H(z) = 1 + \frac{1}{16}z^{-4}$



Since the signal is sampled at 8 kHz the sinusoid at 1kHz will correspond to the  $\omega_1 = 1\text{kHz} \cdot \frac{2\pi}{8\text{kHz}} = \frac{\pi}{4}$  and the

signal at 2kHz will correspond to the  $\omega_2 = 2\text{kHz} \cdot \frac{2\pi}{8\text{kHz}} = \frac{\pi}{2}$

$$\left| H\left(z = e^{j\omega_1} = \frac{\sqrt{2}}{2}(1+j)\right) \right| = \left| 1 + \frac{1}{16}\left(\frac{\sqrt{2}}{2}(1+j)\right)^4 \right| = 0.9375$$

$$\left| H\left(z = e^{j\omega_2} = j\right) \right| = \left| 1 + \frac{1}{16}(j)^4 \right| = 1.0625$$

The IIR filter is a filter with a pole placed over every zero of H:  $H_{IIR}(z) = \frac{1}{1 + \frac{1}{16}z^{-4}}$

The difference equation will be:  $y(n) = x(n) - \frac{1}{16}y(n-4)$

### Ex.2

The final spectrum will extend from  $-M \frac{\pi}{3} \leftrightarrow M \frac{\pi}{3}$  i.e.  $-4 \frac{\pi}{3} \leftrightarrow 4 \frac{\pi}{3}$ , since it extends above  $\pi$ , aliasing

will be present and it can be avoided applying a low pass filter to cut frequencies higher than  $\frac{\pi}{4}$

### Ex.3

```
clc
clear all
close all

% Given the filter h=[0,2,1] and the signal x=[4 3 2 6 4 3];
% with time indices starting both from zero

%1) compute y as the linear convolution of h and x in the time domain
% (you can use MATLAB functions)
h=[0,2,1];
x=[4 3 2 6 4 3];
y=conv(x,h);

%2) compute y2 as the linear convolution of h and x in the frequency domain
% (you can use MATLAB functions)

Nfft=length(h)+length(x)-1;

X=fft(x,Nfft);
H=fft(h,Nfft);
Y2=X.*H;
y2=ifft(Y2,Nfft);

% 3) Plot the squared error between y and y2

figure;
plot(0:Nfft-1,(y-y2).^2);
title('Squared error');
xlabel('n');
ylabel('|y-y_2|^2')
% 4) Plot the two signals y and y2 in the same plot

figure;
plot(0:Nfft-1,y);
hold on
plot(0:Nfft-1,y2,'r--');
hold off
title('Time vs Frequency domain linear convolution');
xlabel('n');
ylabel('y(n), y_2(n)')
legend('Time-domain convolution','Frequency-domain convolution');
```