Multimedia Signal Processing 1st Module

28/11/2013

Ex.1 (Pt.16)

A signal $x(n)$ has been filtered by a linear time-invariant system with impulse response:

$h(n)=\delta(n)-\frac{1}{2}\delta(n-1)$. The output of the system is the distorted signal $y(n)$. Theoretically,

$x(n)$ can be recovered from $y(n)$ through an inverse filter having a system function equal to

the reciprocal of the system function of the distorting filter: $H_i(z)=\frac{1}{H(z)}$.

1) Determine the z-transform $H(z)$ of the impulse response, and plot its zeros-poles plot.
2) Determine analytically $h_i(n)$ [the generating series], is it a FIR or an IIR filter? Plot its amplitude response.
3) If $x(n)=\delta(n)+\delta(n-1)+\delta(n-2)$, calculate $y(n)$ by doing the linear convolution directly.
4) Calculate $y(n)$ working directly in the frequency domain using the DFT.
5) Determine the z-transform of $Y(z)$ and determine the z-transform of $Y(z)\cdot H_i(z)$.

Ex.2 (Pt.6)

The signal $x(n)$, $x(n)=\{\ldots,0,0,\underline{1},2,3,2,1,0,0,\ldots\}$, where $\underline{1}$ represents $x(0)$, is applied as the input to the following system:

$$x(n) \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{H(z)} \longrightarrow \boxed{\downarrow 3} \longrightarrow y(n)$$

If the impulse response $h(n)$ is given by $h(n)=\{\underline{1},2\}$, the what is the output signal $y(n)$?

Using the same impulse response, what will be the output in this case?

$$x(n) \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{H(z^2)} \longrightarrow y(n)$$

Ex.3 (Pt. 11 – MATLAB code)

A LTI filter is defined as: $H(z)=\frac{B(z)}{A(z)}=\frac{1+cz^{-1}}{1-az^{-1}-bz^{-2}}$

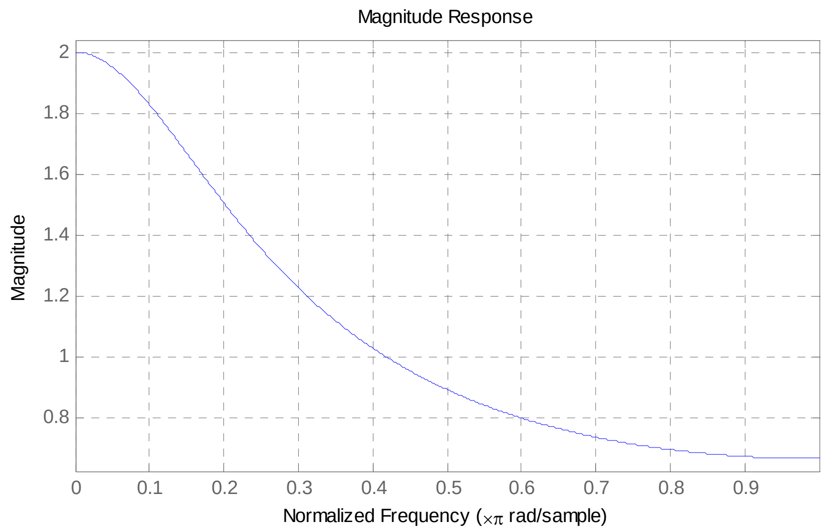
a) Choose randomly a, b, c, but in order to implement a minimum phase stable filter.
b) Compute and plot poles and zeros in the complex plane.
c) Compute and plot the Frequency response.
d) Compute and plot the impulse response h(n) at n = 0, …, 100.
e) Defined x(n) = sin(0.1 π n), compute and plot the output at n=0, …, 100.

Solutions

Ex.1

$$H(z) = 1 - \frac{1}{2} z^{-1}$$

$H_i(z) = \dfrac{1}{1 - \frac{1}{2} z^{-1}}$. It is a IIR stable filter, $h(n) = \left(\frac{1}{2}\right)^n u(n)$



$$y(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{2}\delta(n-2) - \frac{1}{2}\delta(n-3)$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix} \qquad H(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{1}{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 + \frac{1}{2} j \\ \frac{3}{2} \\ 1 - \frac{1}{2} j \end{bmatrix}$$

$$Y(k) = \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} - j \\ \frac{3}{2} \\ \frac{1}{2} + j \end{bmatrix} \qquad y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} - j \\ \frac{3}{2} \\ \frac{1}{2} + j \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$Y(z) = 1 + \frac{1}{2} z^{-1} + \frac{1}{2} z^{-2} - \frac{1}{2} z^{-3}$$

$$Y(z)\cdot H_i(z)=\frac{1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}-\frac{1}{2}z^{-3}}{1-\frac{1}{2}z^{-1}}$$

Ex.2

The outputs will be, in the first case: $\{1,4,2,2\}$ while in the second one will be $\{1,0,4,0,7,0,8,0,5,0,2\}$

Ex.3

```
clc
clear all
close all




% y(n) = ay(n-1) + by(n-2) + x(n) + cx(n-1)
% this is a IIR filter , order 2 .
% transfer functionis :
% Y(z)/X(z) = (1 + czˆ-1) / (1 - azˆ-1 - bzˆ-2)
% this is equal to :
% 1 + czˆ-1 / (1 - ( p1 + p2 ) zˆ-1 + p1 p2 zˆ-2)
% where p1 and p2 are the poles of the transfer function:
%  (z^2 - a z - b )/z^2 = (p1 - z) (p2 - z)/z^2
%  (z^2 - a z - b )/z^2 = (p1 z^-1 - 1) (p2 z^-1 - 1)
%  (z^2 - a z - b )/z^2 = 1 - ( p1 + p2 ) zˆ-1 + p1 p2 zˆ-2)

% Hence we choose |p1| and |p2| < 1 if we want the filter to be stable.
% For instance , p1 = 0.9 and p2 = -0.7 ---> a = 0.2 , b = 0.63.
% Then if we want also a minimun phase filter we choose |c| (the zero)
% inside the unit circle, for example c = 0.8
b = [1 0.8];
a = [1 -0.2 -0.63];
figure, zplane(b,a)

% b
z = roots(b)
p = roots(a)
figure, zplane(z,p)

%without using freqz
%the frequency response is the DTFT of the (truncated) impulse response:
N = 1024;
delta = [1, zeros(1,N-1)];
h = filter(b, a, delta);
figure,
plot(abs(h))



H = fft(h,N);
w = 2*pi*[0 :N-1]/N;
figure,
subplot(2,1,1);
plot (w/pi,20*log10(abs(H))); title('using filter')
axis( [ 0 1 -20 30 ] ) ;
xlabel( 'frequency in pi units');
ylabel( '|H(w)| [ dB ]')
subplot(2,1,2);
plot(w/pi, angle(H)/pi);
axis([ 0 1 -1 1 ]);
xlabel('frequency in pi units');
ylabel('<H(w)in pi units')
```

```
% using freqz
figure,
subplot(2,1,1);
[H, w] = freqz(b,a,N);
plot(w/pi,20*log10(abs(H))); title('using freqz')
axis( [ 0 1 -20 30 ] ) ;
xlabel( 'frequency in pi units');
ylabel( '|H(w)| [ dB ]')
subplot(2,1,2);
plot(w/pi, angle(H)/pi);
axis([ 0 1 -1 1 ]);
xlabel('frequency in pi units');
ylabel('<H(w) in pi units')

% filtering
% input
n=[0:1:100];
x= sin((pi/10)*n);
y=filter(b,1,x);
figure,
plot(abs(y))
```