Ex.1 (Pt.16, 2 points every question)

Consider a causal filter with three zeros in $z = -1$; $z = -j$; $z = j$;

1. Define its zeta transform.
2. Define its impulse response.
3. Draw an approximate amplitude response of the filter and quote it at the null frequency.
4. What kind of filter is it? [low pass, high pass, band pass, band stop]

Apply it to the following <u>periodic</u> signal: (every period is made of the following 4 samples) 1, -2, -2,1

5. What will be the obtained output?
6. Define the $W_{4\times4}$ matrix to perform the DFT.
7. Find the DFT for the input and for the filter.
8. Evaluate the DFT for the output and its Inverse DFT

Ex.2 (Pt.12)

The signal $x(t) = \cos(2\pi 25t)$ is sampled at $200\dfrac{samples}{s}$ and then upsampled of an order of 3 (two zeros added every sample) .

1. (4 points) Represent the spectrum in normalized frequencies before and after upsampling.

   The filter defined at the first exercise is then applied to the upsampled signal.

2. (12 points) What will be the power spectrum (square valued of the amplitude at different frequencies) or the interpolated signal?

Ex.3 (Pt. 6 – MATLAB code)

Solve the Exercise 1 using MATLAB:

Define the filter *h* as define in the text and the periodic signal *x:*(every period is made of the following 4 samples) 1, -2, -2,1.
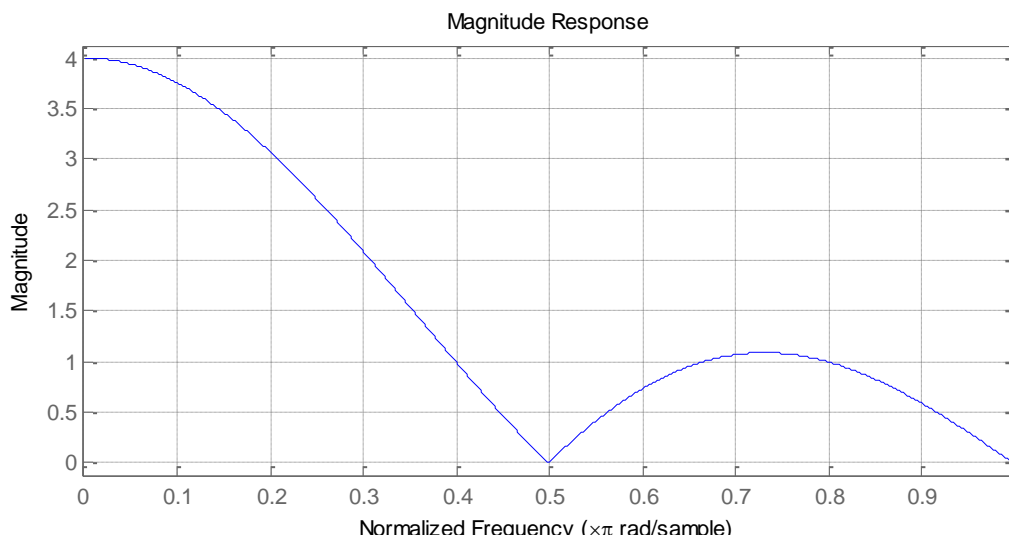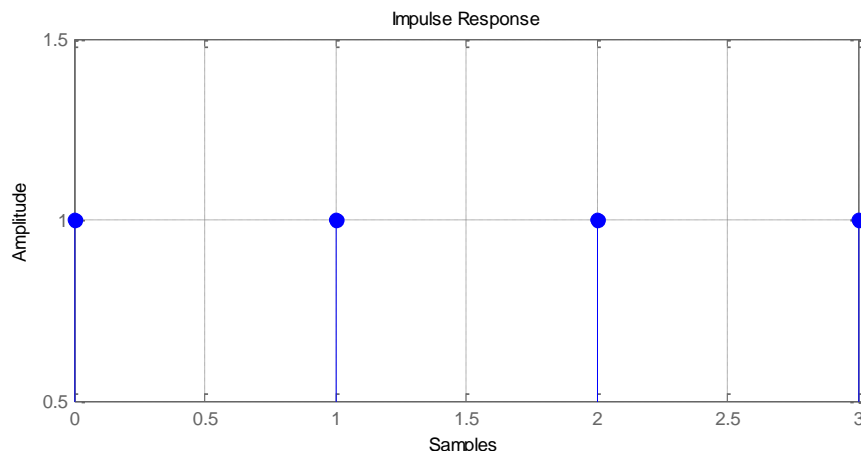Calculate the DFT of the input, the filter and the output.
Represent the magnitude and phase of the three DFT of the aforementioned signals,
Calculate and plot the output signal in the time domain.

Solutions

Ex.1

The zeta transform will be: $H(z)=(1+z^{-1})(1+jz^{-1})(1-jz^{-1})=1+z^{-1}+z^{-2}+z^{-3}$

Impulse Response

(plot: stem plot of Impulse Response, Amplitude vs Samples, values of 1 at samples 0, 1, 2, 3)

Magnitude Response

(plot: Magnitude vs Normalized Frequency (×π rad/sample))

It is a low pass filter

The output of the filtered signal will be [-2, -2, -2, -2].

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

The DFT of the filter will be [4,0,0,0]:

$$DFT(h(n)) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The DFT of the signal will be :

$$DFT\big(x(n)\big)=\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}\begin{bmatrix}1\\-2\\-2\\1\end{bmatrix}=\begin{bmatrix}-2\\3+3j\\0\\3-3j\end{bmatrix}$$

The DFT of the output will be:

$$DFT\big(y(n)\big)=\big[4\cdot(-2),0\cdot(3+3j),0\cdot0,0\cdot(3-3j)\big]=[-8,0,0,0]$$

$$iDFT\big(Y(k)\big)=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\begin{bmatrix}-8\\0\\0\\0\end{bmatrix}=\begin{bmatrix}-2\\-2\\-2\\-2\end{bmatrix}$$

Which is the same result that we got with the circular convolution in the time domain.

Ex. 2

The signal is made of two impulses at $\omega=\pm\dfrac{\pi}{4}$ with amplitude 1. Introducing zeros with the upsampling the

normalized spectrum to the new frequency ($600\dfrac{samples}{s}$) will present the two original impulses that move

at $\omega=\pm\dfrac{\pi}{4\cdot3}=\pm\dfrac{\pi}{12}$ a replica around $\dfrac{2}{3}\pi$: two impulses at $\left(\dfrac{2}{3}-\dfrac{1}{12}\right)\pi=\dfrac{7}{12}\pi$ and $\left(\dfrac{2}{3}+\dfrac{1}{12}\right)\pi=\dfrac{9}{12}\pi$ and

their counterpart at negative pulsations: $-\dfrac{7}{12}\pi$ and $-\dfrac{9}{12}\pi$.

The magnitude at the frequencies $\dfrac{1}{12}\pi,\dfrac{7}{12}\pi,\dfrac{9}{12}\pi$ will be:

$$\left|1+e^{-j\frac{\pi}{12}}+e^{-j\frac{2\pi}{12}}+e^{-j\frac{3\pi}{12}}\right|=\sqrt{\left(1+\cos\frac{\pi}{12}+\cos\frac{2\pi}{12}+\cos\frac{3\pi}{12}\right)^2+\left(-\sin\frac{\pi}{12}-\sin\frac{2\pi}{12}-\sin\frac{3\pi}{12}\right)^2}=3.83$$

$$\left|1+e^{-j\frac{7\pi}{12}}+e^{-j\frac{14\pi}{12}}+e^{-j\frac{21\pi}{12}}\right|=\sqrt{\left(1+\cos\frac{7\pi}{12}+\cos\frac{14\pi}{12}+\cos\frac{21\pi}{12}\right)^2+\left(-\sin\frac{7\pi}{12}-\sin\frac{14\pi}{12}-\sin\frac{21\pi}{12}\right)^2}=0.63$$

$$\left|1+e^{-j\frac{9\pi}{12}}+e^{-j\frac{18\pi}{12}}+e^{-j\frac{27\pi}{12}}\right|=\sqrt{\left(1+\cos\frac{9\pi}{12}+\cos\frac{18\pi}{12}+\cos\frac{27\pi}{12}\right)^2+\left(-\sin\frac{9\pi}{12}-\sin\frac{18\pi}{12}-\sin\frac{27\pi}{12}\right)^2}=1.08$$

So the power spectrum will be: $H^2\left(\omega=\dfrac{1}{12}\pi\right)=14.67$, $H^2\left(\omega=\dfrac{7}{12}\pi\right)=0.40$ $H^2\left(\omega=\dfrac{1}{12}\pi\right)=1.17$

Ex. 3

```
x=[1 -2 -2 1]

h=[1 1 1 1]

Y=fft(x).*fft(h)

figure

subplot(1,2,1);

stem(abs(fft(x)));

subplot(1,2,2);

stem(angle(fft(x)));

figure

subplot(1,2,1);

stem(abs(fft(h)));

subplot(1,2,2);

stem(angle(fft(h)));

figure

subplot(1,2,1);

stem(abs(Y));

subplot(1,2,2);

stem(angle(Y));

y=ifft(Y)

figure;

stem(y);
```