Multimedia Signal Processing
First Module

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1) Consider a signal made of the infinite repetition of the following 4 samples: {0;1;2;3}
    a. Define the DFT matrix and provide the 4 Fourier coefficients for this signal.[4Pt]
    b. An high pass filter, whose z-trasform is $1 - z^{-1}$, is applied to the signal. What will be the samples of the filtered signal? [4Pt]
    c. What will be the DFT of the final signal? [4Pt]

2) The same signal of point 1 is interpolated by an order of 3 (two new samples for every of the original signal).
    a. What will be the Fourier transform when all the added samples have a value of 0? [3Pt]
    b. Define the DFT of an ideal low pass filter for the interpolation of the signal.[3Pt]
    c. If I use a FIR to approximate this filter whose samples are: $\{\left\{\frac{1}{3}; \frac{2}{3}; 1; \frac{2}{3}; \frac{1}{3}\right\}\}$, what will be the output signal in the time domain? [3Pt]
    d. What will be the difference with respect to the result using the ideal filter defined at point b) ? Discuss both in the time and in the frequency domain. [3Pt]

3) [MATLAB] Build a signal sum of three different sinusoids sin(2πft) at pulsations w1= π/8, w2=π/10 w3= π/3. The signal is defined over a temporal axis of 512 samples. (Assume that the sampling period T=1).[3Pt]
Decimate the signal by a factor M=4 using the Matlab function 'fir1' for designing the filter, but not the function 'decimate'. [3Pt]
Compare the original and the decimated signals in the time and in the frequency domain (only modula). [3Pt]

Solutions

ES1

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Using this matrix to obtain the DFT the components at the different frequencies will be:
6 (at frequency 0), -2+2j at ($\omega = \frac{\pi}{2}$), -2 at Nyquist frequency and -2-2j at (($\omega = -\frac{\pi}{2}$).

The samples after filtering will be: 0-3, 1-0, 2-1 and 3-2, i.e. [-3, 1, 1, 1];

Its DFT will be [0, -4, -4; -4]

ES2

The DFT of the new upsampled signal will be (for frequencies from $-\pi$ to $\pi - \frac{\pi}{6}$):-2, -2-2j, 6,-2+2j,

-2, -2-2j, 6,-2+2j, -2, -2-2j, 6,-2+2j, (the original DFT repeated three times)

The DFT of the ideal filter would be: (for frequencies from $-\pi$ to $\pi - \frac{\pi}{6}$): 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0.

The samples, after the linear interpolation will be: 0 1/3, 2/3, 1, 4/3, 5/3, 2, 7/3, 8/3, 3, 2, 1.

Es.3

```
Nfft=1024;
N=512;
M=4;
fc=1/(2*M)


h = fir1(N,2*fc) ;


w1=pi/8;
w2=pi/10;
w3=pi/3;
n=[0:512];
x = sin(w1*n)+sin(w2*n)+sin(w3*n);


%%%%%%%%%%%%%%%%%
%decimate
% filtering
xfilt = conv(h,x);
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```
%downsampling
xdec = xfilt(1:M:end);

figure,
subplot(2,1,1), plot(x)
subplot(2,1,2), plot(xdec)

[H, w] = freqz(x, 1, 1024) ;
[Hdec, w] = freqz(xdec,1,1024);
figure,
w=2*pi*[0:Nfft-1]./Nfft;
subplot(2,1,1), plot(w, 10*log10(abs(H).^2))
subplot(2,1,2), plot(w, 10*log10(abs(Hdec).^2), 'r--');
```