Multimedia Signal Processing - 1st Module – Exam 18/11/2010

ES.1 Consider filter whose z transform is the following:

$$H(z) = \frac{1 - \frac{1}{4} z^{-2}}{1 + \frac{1}{4} z^{-2}}$$

(2pt.) What kind of filter is it? Draw the zero-poles diagram.

(3pt.) Draw the amplitude of the transfer function using the geometric method.

(4pt.) Draw the phase of the transfer function using the geometric method.

(4pt.) The following signal: $x(t) = \cos(2\pi 50t)$ is sampled at a sampling frequency of 100Hz and then filtered by $H(z)$. What will be the output sampled signal $y(n)$?

(2pt.) What will be the group delay for the continuous frequency of the previous signal? What is its meaning?

ES.2 Consider the sequence:

x(n) = cos(0.36 π n) + cos(0.42 π n)

(3pt.) Compute and plot the DFT of x(n) (only the modulus) for 0≤n ≤ 100.

(3pt.) Compute and plot the DFT of x(n) (only the modulus) for 0≤n ≤ 10.

(3pt.) Compute and plot the 100 samples of the DFT of x(n) for 0≤n ≤ 10 (zero padding) (only the modulus). Which is the difference between the plot of the first question and the one of the second question?

ES.3

(4pt.) Implement an all pass filter that has two complex poles in ρ e^(j ω_0) and ρ e^(-j ω_0), with ρ = 0.9 and ω0=0.36 π.

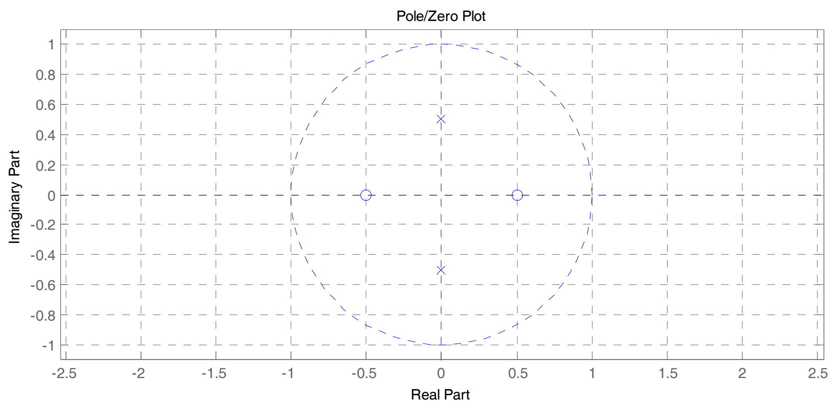
(3pt.) Compute its frequency response and represent its modulus and phase with frequency expressed in π units (Number of samples N=1024).

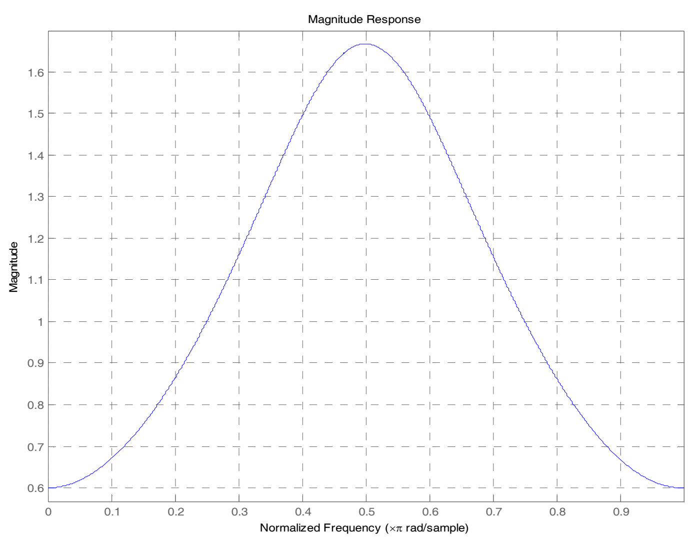
(3pt.) Compute its impulse response.

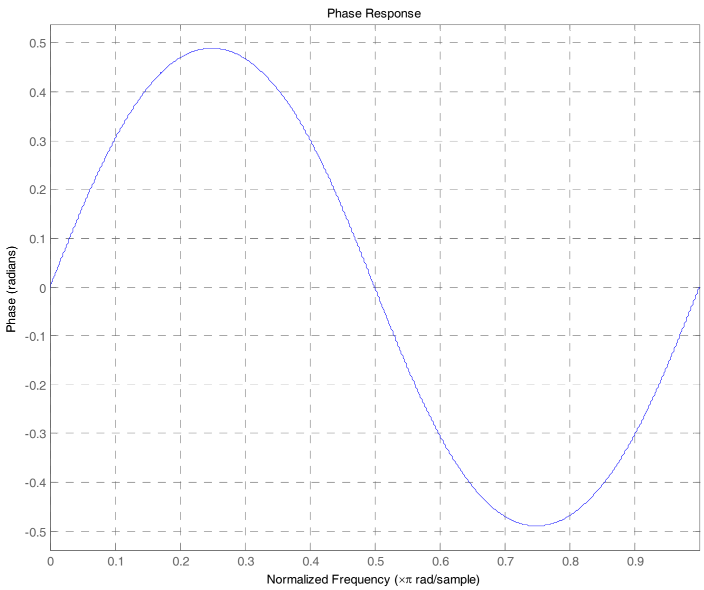
Solutions

Es. 1

It is a band pass filter







The sampled signal corresponds to a sinusoid at the Nyquist frequency of amplitude 1. Since at that frequency the phase contribution of the filter is zero and the filter amplitude is 3/5 (obtained substituting *z*=-1 in the filter equation) the output will be $x(t) = \frac{3}{5}\cos(2\pi 50t)$.

The group delay for the continuous frequency can be calculated as:

$$H(\omega) = \frac{1-\frac{1}{4}e^{-j2\omega}}{1+\frac{1}{4}e^{-j2\omega}} = \frac{4e^{j2\omega}-1}{4e^{j2\omega}+1} = \frac{4\cos(2\omega)-1+4j\sin(2\omega)}{4\cos(2\omega)+1+4j\sin(2\omega)}$$

$$\angle H(\omega)\Big|_{\omega\cong 0} = \tan^{-1}\left(\frac{4\sin(2\omega)}{4\cos(2\omega)-1}\right) - \tan^{-1}\left(\frac{4\sin(2\omega)}{4\cos(2\omega)+1}\right) \cong$$

$$\cong \tan^{-1}\left(\frac{8\omega}{3}\right) - \tan^{-1}\left(\frac{8\omega}{5}\right) \cong \frac{8\omega}{3} - \frac{8\omega}{5} = \frac{16}{15}\omega$$

$$-\frac{\partial \angle H(\omega)}{\partial \omega}\bigg|_{\omega=0} = -\frac{16}{15}$$

Es.2 Solution

SOLUZIONE:

```
n=[0:1:99];

x=cos(0.36*pi*n)+cos(0.42*pi*n);

% a) Spectrum based on the first 100 samples of x(n)

X=fft(x);
k=0:1:99;w=2*pi/100*k;

figure; plot(w/pi,abs(X));  title('DTFT Magnitude');
xlabel('frequency in pi units')

% b) Spectrum based on the first 10 samples of x(n)

n1=[0:1:9];
y1=x(1:1:10);
Y1=fft(y1);
k1=0:1:9; w1=2*pi/10*k1;

figure; stem(w1/pi,abs(Y1));   title('Samples of DTFT Magnitude');
xlabel('frequency in pi units')

% c) High density spectrum (100 samples) based on the first 10 samples of x(n)

n2=[0:1:99];
y2=[x(1:1:10) zeros(1,90)];

Y2=fft(y2);
k2=0:1:99;  w2=2*pi/100*k2;

figure;  plot(w2/pi,abs(Y2));   title('DTFT Magnitude');
xlabel('frequency in pi units')
```

Es. 3

```
% a) All pass filter with two complex pole in rho*exp(j*omega0) and
% rho*exp(-j*omega0)

rho = 0.9;
omega0 = 0.36*pi;

p1 = rho*exp(j*omega0);
p2 = rho*exp(-j*omega0);

a = poly([ p1; p2]);  % denominator
b = conj(fliplr(a));

figure, zplane(b,a)

% b) Frequency response
N=1024;
[H, w] = freqz(b,a,N);

figure, subplot(2,1,1), plot(w/pi,abs(H));
title('Module of the frequency response'),
xlabel('frequency in pi units'); ylabel('|H(\omega)|');

subplot(2,1,2), plot(w/pi, phase(H)/pi)
title('Phase of the frequency response'),
xlabel('frequency in pi units'); ylabel('\angle(H(\omega))');


% c) Impulse response
delta = [ 1 ; zeros(N-1,1)]';
h = filter(b,a,delta);
figure, stem([0:50],h(1:51)), title('impulse response')
```