1. 04.12.2007

Let $h_1(n)$ and $h_2(n)$ denote the impulse responses of two LTI digital filters

$$h_1(n) = a(-0.9)^n u(n) \tag{1}$$

$$h_2(n) = \delta(n-1) \tag{2}$$

where $u(n)$ denotes the step function and $a \in \mathbb{C}$.

Let $h_a(n)$ and $h_b(n)$ denote the following impulse responses:

$$h_A(n) = h_1(n) + h_2(n) \tag{3}$$

$$h_B(n) = h_1(n) * h_2(n) \tag{4}$$

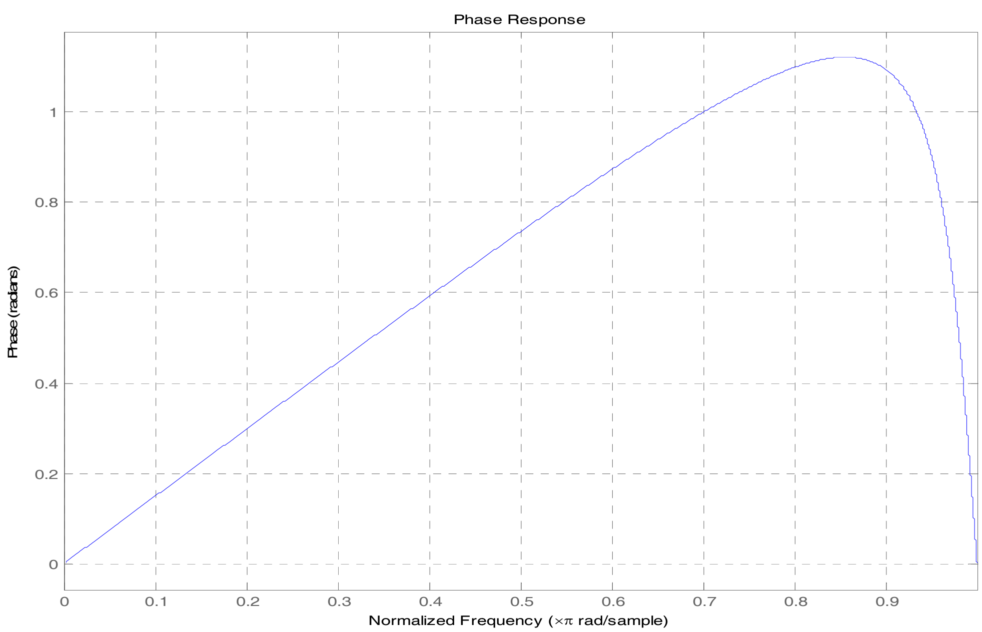
- Find the transfer functions $H_1(z)$ and $H_2(z)$.
- Find the transfer functions $H_A(z)$ and $H_B(z)$.
- Find the value of the parameter a such that the frequency response satisfies the constraint

  $$H_A(e^{j\omega})|_{\omega=0} = 0 \tag{5}$$

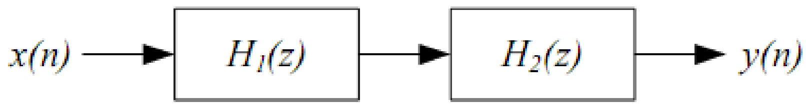
  (Ignore the trivial solution $a = 0$).
- Plot the position of the poles and zeros of $H_A(z)$ and $H_B(z)$ in the complex plane.
- Sketch the frequency response of the modulus of $H_A(z)$ and $H_B(z)$.
- Find the values of $H_A(e^{j\omega})$ and $H_B(e^{j\omega})$ for $\omega = 0, \pi$.
- For each of the filters $H_1(z)$, $H_2(z)$, $H_A(z)$, $H_B(z)$, state if they have linear phase.

I do not know how to solve the last question. Since H1(z),Ha(z) are not all pole filters, which is not in the form of $1+az^{-1}+bz^{-2}$....

The filter h1 is a one-pole filter but the phase will not be linear:

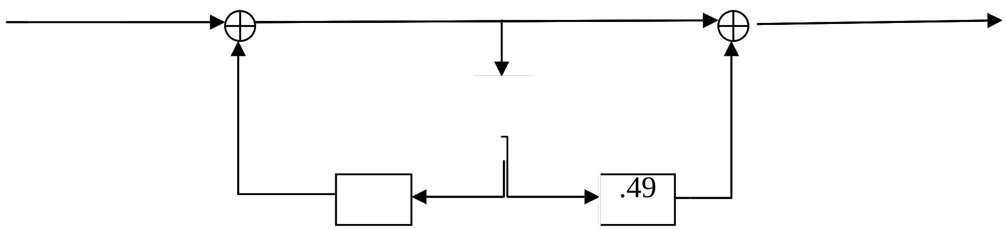


The filter h2, instead, is a 1-pole filter (z^-1) but the pole is in the origin and its phase will be linear.
Obviously their sum will not have linear phase and neither their product since you are adding or multiplying an object with a linear phase with one with a non linear one.



2. 23/11/2009
1) EX1 question 1 ask to define a block scheme in terms of inputs x(n) and y(n) when knowing a certain H(z). Is it just like the above?

No, should be something like:



In question 2, what does it mean when saying different normalized frequencies? Is it mean, when the phase is 0, pi/2, pi....
No, it means that the considered frequencies are divided by the sampling frequency and so are in the range –½ and ½ the two normalized nyquist frequencies.

2)EX3 ask us to represent the likelihood function knowing 3 realization and pdf of variable X.
I do not know how to solve this kind of question.

Accordingly to the definition of the likelihood function, since the rectangle width is 1, there is only one possibility for the pdf to be fitted to the distribution. Its average must be in 2 and the likelihood function will be an impulse in 2. While in the second question there are more opportunities and the likelihood function will be a rectangle that extends from 1.9 to 2.1 since all uniform distribution of width 1, centered in a value in a range between 1.9 and 2.1 will satisfy all the 3 samples.

3)In EX4, I know how to calculate r(0),r(1), but I do not know to to figure out r(3)

Just continue using Y-W equations extending the matrix:
The last raw will be [r(3) r(2) r(1) r(0)] that will be multiplied by vector [1 a1 0 0].
3. When H(z)is given, how can I decide the value M, when asked to design a filter for Interpolation and Decimation?
One such question is in

21/09/2007 EX2 Question 2

- illustrate the frequency domain effect of downsampling and upsampling.
- Let $h(n)$ be a FIR filter be defined by the following transfer function

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6} \quad (1)$$

  - What is the phase delay of the filter?
  - Show a block diagram that illustrates polyphase decimation, where $h(n)$ is used as anti-aliasing filter.

It depends from case to case, anyway is a common rule to use the first zero of the filter as the new sampling frequency after decimation (twice the nyquist freq.) as for the second question, while for interpolation there is no limit.