Ex.1 (Pt.12)

A signal $x(t)$ is sampled at 8kHz. We want to upsample it to 10 kHz.

1. [3 pts.] Describe and representthe processing chain in order to get the proper upsampled signal. [Provide numerical values of the parameters at every step]

The initial sequence $x[n]=\{-1,2,1,2\}$ is sampled at 8kHz and we have a FIR low pass filter $h[n]=\{1,\sqrt{2},2,\sqrt{2},1\}$.

2. [9 pts.] Provide the output, $x'[n]$, i.e. the final upsampled signal at the end of the whole process.

Ex.2 (Pt.12)

From the following signal $x[n]=\{3,-2,-2,5,1,1\}$, that was sampled at 12kHz, we need to remove completely the spurious components at 2kHz and at 4kHz, preserving the other ones.

1. [3pts.] Working only in the Frequency domain, provide the $\mathbf{W}$ matrix in order to get the DFT of the signal.
2. [5pts.] Find the DFT of the signal, define and apply the proper filter to remove justthe spurious components preserving the other ones.
3. [4 pts.] Find the final output signal $y[n]$ in the time domain.

Ex.3 (Pt. 11 – MATLAB code)

Suppose you have to create the MATLAB script 'exam.m'.

1. [1 pt] Which are the lines of code the script should begin with in order to close the opened figures, clear the workspace and clear the command window?
2. [6 pt] You are given a LTI system characterized by the following finite-difference equation:

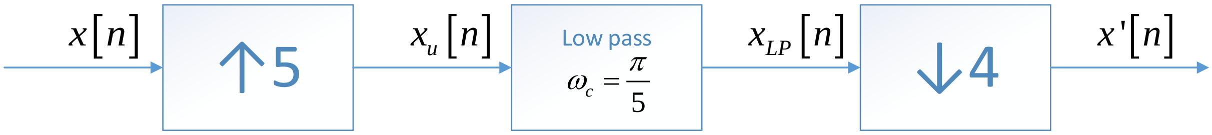
   y(n) = 2x(n) – 2√2x(n-1) + 2x(n-2) + √2/2y(n-1) – 0.25y(n-2)

   . Write the transfer function of the filter in Z-domain, as H(z) = B(z) / A(z)
   a. Define B(z) and A(z) as arrays in MATLAB
   b. Evaluate the value of the filter h(n) in n = 0 without converting the filter to time domain
   c. Evaluate the poles and the zeros
   d. Plot zeros and poles in the Z plane
   e. Write a MATLAB function 'is_stable.m' which receives as input B(z) and A(z) of a generic filter and returns:
      i. 1 if the system is stable
      ii. -1 if the system is unstable
   f. Test the function 'is_stable.m' on the filter H(z) defined above, assigning to the variable 'stability' the output of the function.
3. [4 + 1 extra pt] Given the sinusoidal signal x, sampled at Fs = 1.6KHz, with amplitude 1.5, frequency 200 Hz, duration 1.3 seconds

   . Filter the signal x with the filter H(z) defined above.
   a. Plot the magnitude of the DFT of the filtered signal as a function of normalized frequencies defined between [0, 1).
   b. [1 extra pt] What do you expect to see in the DFT of the initial signal x and in DFT of the filtered signal?

Solutions

Ex.1



The upsampled signal will be:

$$x_u[n]=\frac{1}{5}\{-1,0,0,0,0,2,0,0,0,0,1,0,0,0,0,2,0,0,0,0\}.$$

Applying the low pass filter we will get:

$$x_{LP}[n]=\frac{1}{5}\left\{-1,-\sqrt{2},-2,-\sqrt{2},-1,2,2\sqrt{2},4,2\sqrt{2},2,1,\sqrt{2},2,\sqrt{2},1,2,2\sqrt{2},4,2\sqrt{2},2\right\}$$

Applying the downsampling we will get:

$$x'[n]=\frac{4}{5}\left\{-1,-1,2\sqrt{2},2,2\sqrt{2}\right\}$$

The coefficient 4/5 is due to preserve the signal power at the different sample rate.

Ex.2

$$w_6=\frac{2\pi}{6}=\frac{\pi}{3}$$

$$\mathbf{W}=\begin{bmatrix} w_6^{0\cdot 0} & w_6^{0\cdot 1} & w_6^{0\cdot 2} & w_6^{0\cdot 3} & w_6^{0\cdot 4} & w_6^{0\cdot 5} \\ w_6^{1\cdot 0} & w_6^{1\cdot 1} & w_6^{1\cdot 2} & w_6^{1\cdot 3} & w_6^{1\cdot 4} & w_6^{1\cdot 5} \\ w_6^{2\cdot 0} & w_6^{2\cdot 1} & w_6^{2\cdot 2} & w_6^{2\cdot 3} & w_6^{2\cdot 4} & w_6^{2\cdot 5} \\ w_6^{3\cdot 0} & w_6^{3\cdot 1} & w_6^{3\cdot 2} & w_6^{3\cdot 3} & w_6^{3\cdot 4} & w_6^{3\cdot 5} \\ w_6^{4\cdot 0} & w_6^{4\cdot 1} & w_6^{4\cdot 2} & w_6^{4\cdot 3} & w_6^{4\cdot 4} & w_6^{4\cdot 5} \\ w_6^{5\cdot 0} & w_6^{5\cdot 1} & w_6^{5\cdot 2} & w_6^{5\cdot 3} & w_6^{5\cdot 4} & w_6^{5\cdot 5} \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{\pi}{3}} & e^{-j\frac{2\pi}{3}} & -1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{5\pi}{3}} \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} & 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{\pi}{3}} & e^{j\frac{2\pi}{3}} & -1 & e^{j\frac{4\pi}{3}} & e^{j\frac{5\pi}{3}} \end{bmatrix}$$

$$X[k]=\mathbf{W}\cdot\begin{bmatrix} 3 \\ -2 \\ -2 \\ 5 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 6 \\ \dots \\ \dots \\ -2 \\ \dots \\ \dots \end{bmatrix}, H[k]=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The "…" represents a value that does not need to be computed since the filter will set that value to zero. The filter will just preserve the continuous and the Nyquist component at 6kHz.

$$Y[k] = H[k].*X[k] = \begin{bmatrix} 6 \\ 0 \\ 0 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

$$y[n] = iDFT(Y[k]) = \mathbf{W}^{-1}Y[k] = \frac{1}{6}\mathbf{W}^{T}Y[k] = \frac{1}{6}\begin{bmatrix} 6-2 \\ 6+2 \\ 6-2 \\ 6+2 \\ 6-2 \\ 6+2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 4/3 \\ 2/3 \\ 4/3 \\ 2/3 \\ 4/3 \end{bmatrix}$$

Ex.3

```
%% 1.

closeall
clearvars
clc

%% 2.
% Define B(z) and A(z) as arrays in MATLAB

B = [2, -2*sqrt(2), 2];
A = [1, -sqrt(2)/2, 0.25];

% Evaluate the value of the filter h(n) in n = 0
% without converting the filter to time domain

h_0 = B(1) / A(1);

% poles and zeros

zeroes = roots(B);
poles = roots(A);

% zeros and poles in the Z plane

figure;
zplane(B, A);

% MATLAB function ‘is_stable.m’ receives
% as input B(z) and A(z) of a generic filter and returns:
%i.    1 if the system is stable
% ii.   -1 if the system is unstable
% NB: functions should be defined in different files or at the end of
the
% script --> check the end of this script

% Test the function ‘is_stable.m’ on the filter H(z) defined above

stability = is_stable(B, A);

%% 3.

% Given the sinusoidal signal x, sampled at Fs = 1.6KHz,
% with amplitude 1.5, frequency 200 Hz, duration 1.3 seconds

ampl = 1.5;
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```
f0 = 200;
Fs = 1.6e3;
duration = 1.3;
time = 0:1/Fs:duration;

x = ampl * cos(2*pi*f0*time);

% Filter the signal x with the filter H(z) defined above

y = filter(B, A, x);


% Plot the magnitude of the DFT of the filtered signal
% as a function of normalized frequencies between [0, 1).

Yf = fft(y);
N_samples_fft = length(y);
norm_freq_axis = 0: 1/N_samples_fft:(N_samples_fft- 1)/ N_samples_fft;

figure;
plot(norm_freq_axis, abs(Yf));

% What do you expect to see in the DFT of the initial
% signal x and in DFT of the filtered signal?

% the input signal x is a cosine --> we expect to see 2 peaks in
% normalized frequency = 200Hz/1600Hz --> one peak in 1/8 = 0.125
% and the other peak in 1 - 1/8 = 0.875
% the output signal y is the filtered version of x.
% the filter H(z) is a notch filter and has zeros at omega = pi/4,
which
% corresponds to normalized frequency = 1/8... therefore, the sinusoid
is
% canceled by the filter. We expect an almost flat spectrum

%% function code

function [stability] = is_stable(B, A)

% compute the poles of the filter
% NB: zeros are not associated with stability
poles = roots(A);

% check whether any pole is outside the unit circle
if any(abs(poles) > 1)
stability = -1;
else
stability = 1;
end

end
```