

MultimediaSignal Processing 1st Module and Fundamentals of Multimedia Signal Processing

date: 10 September 2019

Ex.1 (Pt.14)

A signal, $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$, with $f_1 = 5\text{kHz}$ and $f_2 = 12\text{kHz}$ is sampled at 60kHz and then, its digital version $x[n]$, is downsampled of an order of 3.

In order to perform the downsampling we just have a single digital IIR filter whose Discrete Difference

Equation is the following: $y[n] = A(x[n] + \sqrt{2}x[n-1] + x[n-2]) + \frac{1}{2}y[n-1]$

With $A = \frac{1}{2(2 + \sqrt{2})}$.

- [5pt] Write the z-transform $H(z)$ of the filter and draw the pole-zero plot.
- [2pt] Represent the amplitude response of the filter in the $0 - \pi$ normalized frequency range.
- [3pt] Represent the signal in the normalized frequencies domain before and after the downsampling, highlighting the aliased components.
- [4pt] Evaluate, in the final signal, the relative amplitude of the aliased components with respect to the main ones.

Ex.2 (Pt.8)

A signal $x[n] = \{-1, -1, 2, 0, 0, 1, -2, 1, 0, 1, 1, -1\}$ shall be filtered with this FIR filter: $h[n] = \{1, -1, 2\}$.

We need to process blocks of 4 samples of the original signal for real time processing and we want to get the result depicting all the intermediate steps with these methods:

- [4pt] Overlap and Add
- [4pt] Overlap and Save

Ex.3 (Pt. 11 - MATLAB code)

1. [2pt.] Generate five cosine tones with the following parameters:

	Amplitude	Frequency [Hz]	Phase [deg]
x1	1.0	220	0
x2	0.75	440	45
x3	0.5	660	90
x4	0.25	880	135
x5	0.125	1100	180

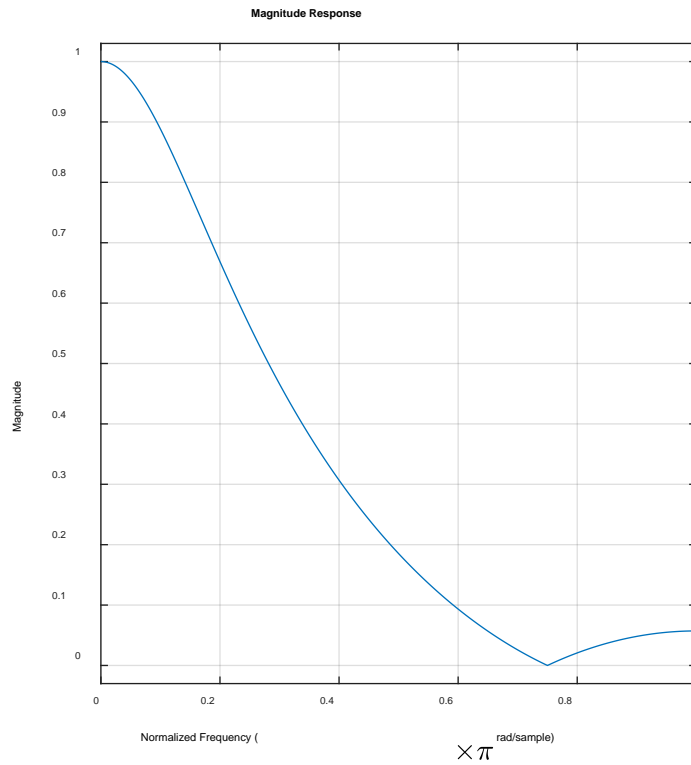
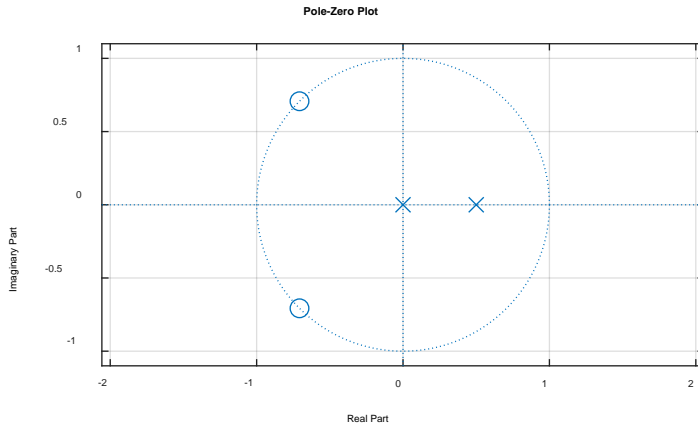
All five signals have a duration of 1 second and a sampling rate of 8000 Hz

2. [1pt.] Generate the signal x6 as the sum of the five signal generated in point 1
3. [2pt] Compute y6 as x6 filtered with a low-pass filter with normalized cut-off frequency of 0.4 and 64 weights
4. [2pt.] Apply a Hanning window to select the first 512 samples of the signals x6 and y6.
5. [1pt.] Compute the DFT of the two windowed x6 and y6 using the MATLAB function `fft`
6. [1pt.] Plot the magnitude of the two DFTs computed at step 5, with frequency in Hz in the x-axis.
7. [2pt.] Comment on what you expect from this result.

Solutions

Ex.1

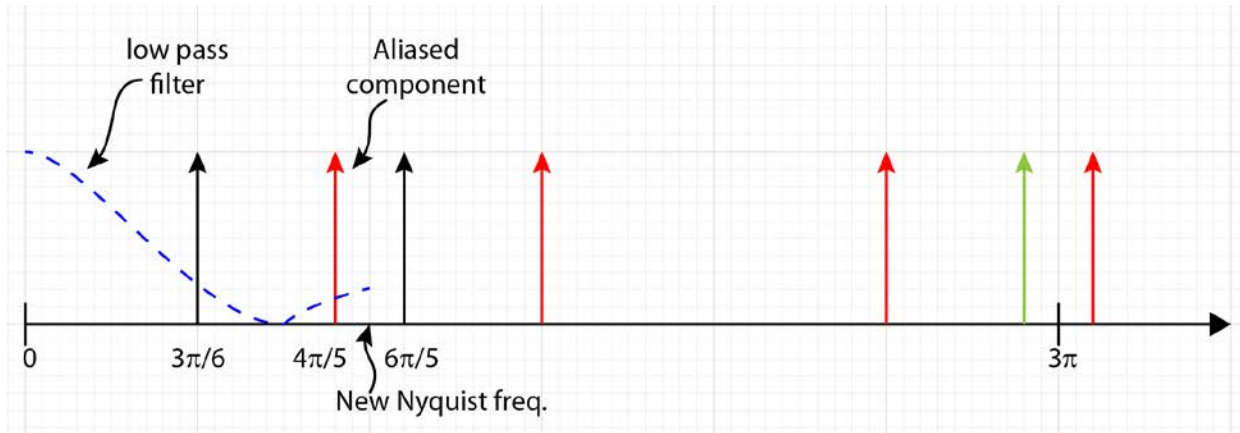
$$H(z) = A \frac{1 + \sqrt{2}z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1}}$$



Normalized frequencies and pulsations of the signal before downsampling



Normalized pulsations after downsampling: in red and green the spectral replicas of the signal due to downsampling. The aliased component is also indicated.



The amplitude of the output for the first sinusoid will be:

$$\left| H(z = e^{j\pi/2}) \right| = \left| A \frac{1 + \sqrt{2}z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1}} \right| = \left| A \frac{1 - \sqrt{2}j - 1}{1 + j/2} \right| = 0.185$$

While for the aliased replica at $2\pi - \frac{6}{5}\pi = \frac{4}{5}\pi$ the amplitude is:

$$\left| H(z = e^{j4\pi/5}) \right| = \left| A \frac{1 + \sqrt{2}e^{-j4\pi/5} + e^{-j8\pi/5}}{1 - \frac{1}{2}e^{-j4\pi/5}} \right| = 0.02$$

Ex.2

Subdividing the signal in 3 blocks for the Overlap and Add we get:

$$x_1[n] = \{-1, -1, 2, 0\}$$

$$x_2[n] = \{0, 1, -2, 1\}$$

$$x_3[n] = \{0, 1, 1, -1\}$$

Filtering each block we get:

$$y_1[n] = \{-1 \ 0 \ 1 \ -4 \ 4 \ 0\}$$

$$y_2[n] = \{0 \ 1 \ -3 \ 5 \ -5 \ 2\}$$

$$y_3[n] = \{0 \ 1 \ 0 \ 0 \ 3 \ -2\}$$

Considering the overlapping tails:

$$\begin{array}{cccccccccccc} | & -1 & 0 & 1 & -4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ | & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 5 & -5 & 2 & 0 & 0 & 0 \\ | & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 3 & -2 \end{array}$$

We get as output:

$$y[n] = \{-1 \ 0 \ 1 \ -4 \ 4 \ 1 \ -3 \ 5 \ -5 \ 3 \ 0 \ 0 \ 3 \ -2\}$$

For the Overlap and Save we need to add two '0's at the beginning to account for the circular convolution effect, the number of zeros added at the beginning of the sequence is the length of the filter minus 1, i.e. $3-1=2$

$$x_1[n] = \{0 \ 0 \ -1 \ -1 \ 2 \ 0\}$$

$$x_2[n] = \{2 \ 0 \ 0 \ 1 \ -2 \ 1\}$$

$$x_3[n] = \{-2 \ 1 \ 0 \ 1 \ 1 \ -1\}$$

Filtering using the circular convolution we get:

$$y_1[n] = \{4 \ 0 \ -1 \ 0 \ 1 \ -4\}$$

$$y_2[n] = \{-3 \ 0 \ 4 \ 1 \ -3 \ 5\}$$

$$y_3[n] = \{1 \ 1 \ -5 \ 3 \ 0 \ 0\}$$

In order to remove the spurious components due to the circular convolution we remove the first two samples of every output sequence; the final output sequence will then be:

$$y[n] = \{-1 \ 0 \ 1 \ -4 \ 4 \ 1 \ -3 \ 5 \ -5 \ 3 \ 0 \ 0 \ \}$$

That is equal to the previous one apart from the last two samples (the tail of the Overlap and Add) that are not present due to the circular convolution approach.

Ex.3

```
% Ex.3 (Pt. 11 - MATLAB code)
% 1) Generate five cosine tones with the following parameters:
% Amplitude Frequency [Hz] Phase [deg]
% x1 1.0 220 0
% x2 0.75 440 45
% x3 0.5 660 90
% x4 0.25 880 135
% x5 0.125 1100 180
% All five signals have a duration of 1 second and a sampling
rate of 8000 Hz
```

```
Fs=4000;
t=0:1/Fs:1;
```

```

X=zeros(5,length(t));
freqs=[220,440,660,880,1100];
phases=[0,45,90,135,180];
ampl=[1,.75,.5,.25,.125];
for i=1:length(freqs)
    X(i,:)=ampl(i)*cos(2*pi*freqs(i)*t+phases(i)/pi);
end

%2) Generate the signal x6 as the sum of the five signal
generated in point 1
x6= sum(X);

% 3) Compute y6 as x6 filtered with a low-pass filter with
normalized cut-off frequency of
%    0.4 and 64 weights
Nf=64;
w_norm=0.4;
b=firl(Nf-1,w_norm);
y6=filter(b,1,x6);

% 4) Apply a Hanning window to select the first 512 samples of
the signals x6 and y6.

win=window(@hanning,512);
x6_win=x6(1:512).*(win. ');
y6_win=y6(1:512).*(win. ');

%5) Compute the DFT of the two windowed x6 and y6 using the
MATLAB function fft

X6=fft(x6_win);
Y6=fft(y6_win);

% 6) Plot the magnitude of the two DFTs computed at step 5,
... with frequency in Hz in the x-axis.
f=linspace(-Fs/2,Fs/2,512);
figure;
subplot(2,1,1);

plot(f, fftshift(abs(X6)));
xlabel('f [Hz]');
ylabel('|X_6(k)|');
subplot(2,1,2);
plot(f, fftshift(abs(Y6)));
xlabel('f [Hz]');
ylabel('|Y_6(k)|');

% 7) Comment on what you expect from this result.

% I'm filtering at 0.4 w.r.t. to Fs=4000,
...so omega =2000Hz, and cut-off frequency is
... 0.4*2000=800 Hz. For this reason, in Y6 I will
... only see up to x3 sinusoids

```