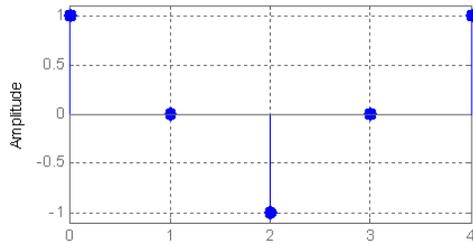
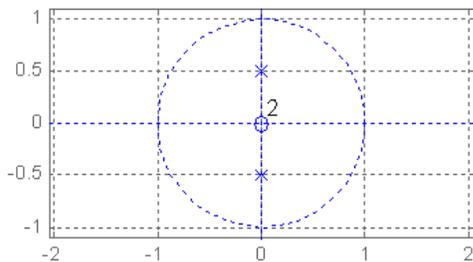


**Ex.1 (Pt.11)**

Consider a digital filter system is composed of a cascade of two filters, the first one has the following impulse response:



While the second filter has two complex conjugate poles in these positions:



- 1) Define the whole zero-pole plot.
- 2) Is it stable or unstable? What kind of filter is it?
- 3) Plot an approximate amplitude response.
- 4) Provide the difference equation of the filter and evaluate the first five samples of the whole filter impulse response.

**Ex.2 (Pt.11 NOT to be done in MATLAB)**

Let  $x(n)$  be a discrete-time rectangular pulse of length  $L = 5$ , i.e.  $x(n) = u(n) - u(n - 5)$ , and  $h(n)$  be a discrete-time rectangular pulse of length  $M = 3$ , i.e.  $h(n) = u(n) - u(n - 3)$ .

The output  $y(n)$  is evaluated as an 8-points sequence obtained from the inverse DFT of the product

$$Y_N(k) = X_N(k)H_N(k).$$

- 1) Define how can be obtained the 64 coefficients of the **W** matrix to get the DFT of  $x(n)$  and  $h(n)$ : do not list all coefficients but only the formula in order to get them where  $r$  and  $c$  represent row and column index of the matrix element.
- 2) Get the 8 values of  $y(n)$  working in the time domain,
- 3) Will time-aliasing be present?

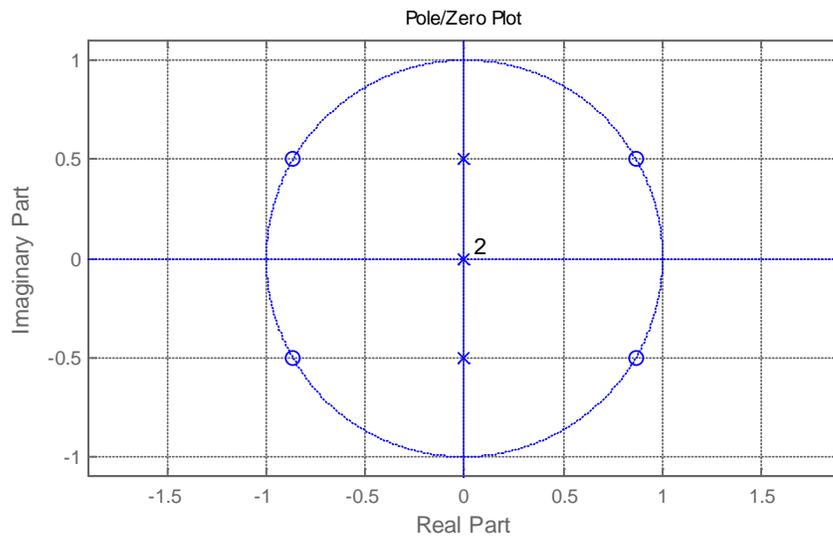
**Ex.3 (Pt. 11 – MATLAB code)**

- 1) implement an all-pass filter that has two complex zeros in  $\rho e^{\pm j\omega_0}$  and with  $\rho = 1.2$  and  $\omega_0 = 0.4\pi$  and two poles (place them in the proper position in order to get an “all-pass” filter).
- 2) plot the poles and the zeros of the all pass filter in the z-plane
- 3) compute its frequency response from 0 to  $\pi$  and plot its modulus and phase in the same figure (two subplots), with the frequency in pi units
- 4) compute its impulse response (the first 512 samples)

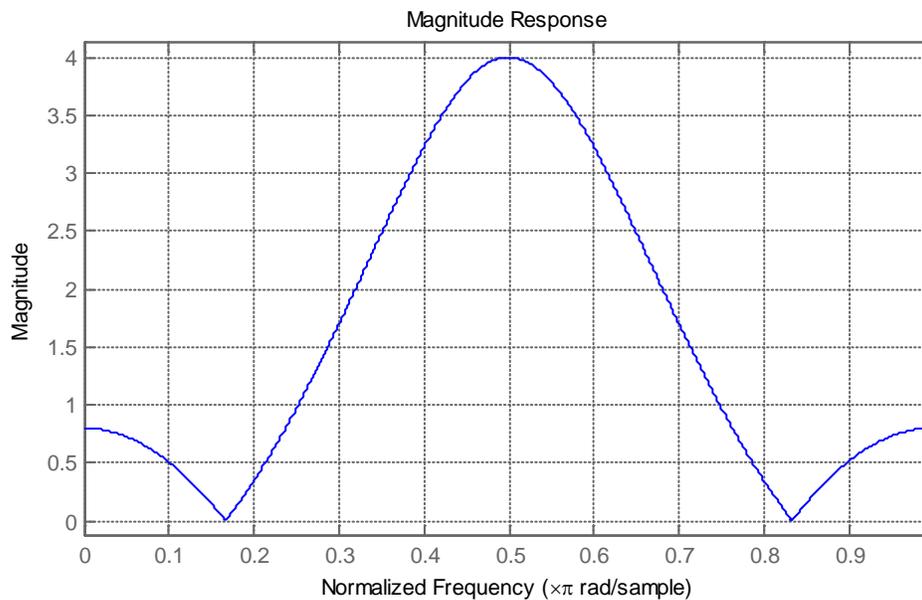
## Solutions

### Ex.1

The filter is a stable band pass filter.



Amplitude response:



$$\text{Difference equation } y(n) + \frac{1}{4}y(n-2) = x(n) - x(n-2) + x(n-4)$$

Impulse response:

$$y(0)=x(0)=1;$$

$$y(1)=0;$$

$$y(2)=-y(0)/4 - x(0)=-5/4$$

$$y(3)=0$$

$$y(4)=-y(2)/4+x(0)=21/16.$$

### Ex.2

The linear convolution of  $x(n]$  and  $h(n]$  will have a length of  $5+3-1=7$ , so, since  $N=8$  there will be no time-domain aliasing.

The DFT coefficient will be  $e^{-j\frac{2\pi}{8}r \cdot c}$

The output, convolving the x and h will be:  $\{1, 2, 3, 3, 3, 2, 1, 0\}$

### Ex.3

```
clc
clear all
close all

% 1) implement an all-pass filter that has two complex zeros in
% rho*e^(+- j omega_0)
rho=1.2;
omega_0=0.4*pi;
z=[rho*exp(-1i*omega_0); rho*exp(+1i*omega_0)];
p=1./conj(z);

% 2) plot the poles and the zeros of the all pass filter in the z-plane
figure;
zplane(z,p);
title('All-pass Filter');

%3) compute its frequency response from 0 to ? and plot its modulus and
% phase in the same figure (two subplots), with the frequency in pi units

a=poly(p);
b=poly(z);
[H, f]=freqz(b,a);
figure;
subplot(2,1,1);
plot(f/pi,abs(H));
title('Magnitude');
xlabel('\omega [\pi units]');
ylabel('|H(z)|');
subplot(2,1,2);
plot(f/pi,unwrap(angle(H)));
title('Phase');
xlabel('\omega [\pi units]');
ylabel('\angle H(z)');

%4) compute its impulse response (the first 512 samples)

N=512;
delta=zeros(N,1);
delta(1)=1;
h=filter(b,a,delta);
figure;
plot(0:N-1,h);
title('Impulse response');
xlabel('n');
```

```
ylabel('h(n)');
```