Multimedia Signal Processing 1st Module

1 /3/2013

Ex.1 (Pt.12)

A signal sampled at 48kHz, is filtered with the following system made of the sequence of filters F1 and F2 with the following outputs:

$$F1: y(n) = x(n) - x(n-1) - \frac{4}{9} y(n-2)$$

$$F2: y(n) = x(n) - \frac{2}{3} y(n-1)$$

The two filters are applied in cascade one after the other.

1. Define the z-transform of the every single filter, F1, F2 and of their cascade (F3).
2. Plot the zeros-poles diagram of the whole filter (F3).
3. Draw an approximate representation of the output amplitude of the filter F3.

A sinusoid of 12 kHz with amplitude 10 enters the system.

4. What will be the amplitude of the output sinusoidal?

Ex.2 (Pt.12)

Given a process with as input a gaussian white noise w_n with $E[w_n] = 0$, and $\sigma^2 = 1$

The output is $x_n = w_n - \frac{1}{2} w_{n-1} + w_{n-2}$

1. Compute the first 3 samples $[r_0, r_1, r_2]$ of the autocorrelation of the process x_n
2. Provide a Parametric spectral estimation of the AR process of order 1 ($H_{AR(1)}$)

Ex.3 (Pt. 11 – MATLAB code)

Given an IIR filter with:

three poles: $p_1 = 0.99$ $p_2 = 0.9\, e^{j\frac{\pi}{8}}$ $p_3 = 0.9\, e^{-j\frac{\pi}{8}}$

three zeros: $z_1 = -1$ $z_2 = e^{j\frac{\pi}{16}}$ $z_3 = e^{-j\frac{\pi}{16}}$

1. Plot poles and zeros in the complex plane.
2. Compute the coefficients of its difference equation.
3. Compute and plot (positive and negative frequencies) the Frequency response using ‘freqz’.
4. Compute and plot the impulse response h(n) at n = 0, …, 10024.
5. Defined x(n) = sin(0.1 π n), compute and plot the output at n=0, …, 10024.

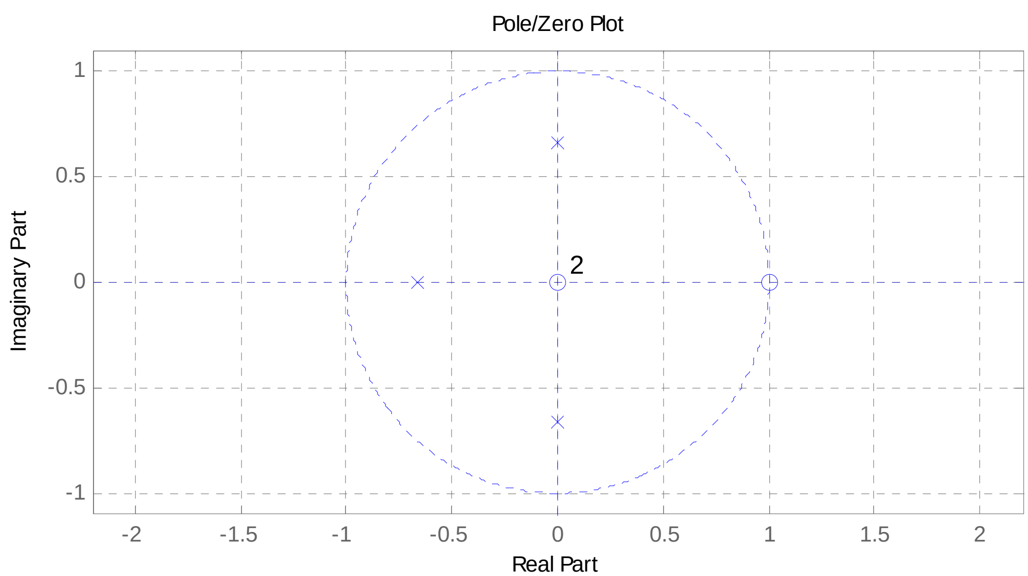
Solutions

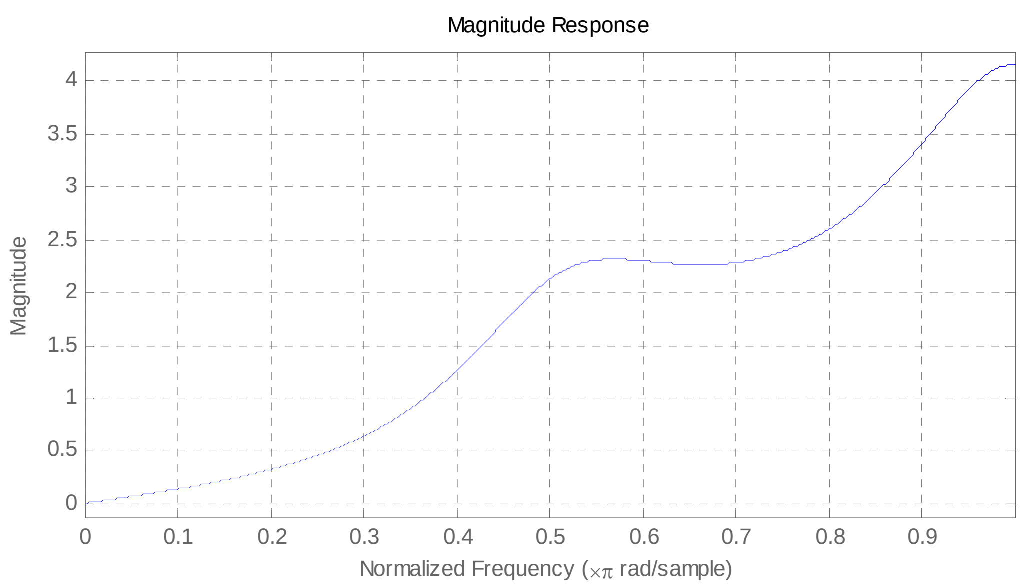
Ex.1

$$Y_1(z) = \frac{1 - z^{-1}}{1 + \frac{4}{9} z^{-2}}$$

$$Y_2(z) = \frac{1}{1 + \frac{2}{3} z^{-1}}$$

$$Y_3(z) = Y_1(z) \cdot Y_2(z) = \frac{1 - z^{-1}}{1 + \frac{2}{3} z^{-1} + \frac{4}{9} z^{-2} + \frac{8}{27} z^{-3}}$$





The amplitude will be, since the normalized pulsation will be $\pm\pi / 2 \rightarrow z = \pm i$, if the input amplitude is 10, 21.2

Ex.2

$$r_0 = 1^2 + \left(-\frac{1}{2}\right)^2 + 1^2 = \frac{9}{4}$$

$$r_1 = \left(-\frac{1}{2}\right)\cdot 1 + \left(-\frac{1}{2}\right)\cdot 1 = -1$$

$$r_2 = 1$$

Applying Yule-Walker formula:

$$\begin{bmatrix} \frac{9}{4} & -1 \\ -1 & \frac{9}{4} \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sigma^2{}_{AR(1)} \\ 0 \end{bmatrix}$$

$$a_1 = \frac{4}{9}$$

$$\sigma^2{}_{AR(1)} = \frac{65}{81}$$

$$H_{AR(1)} = \frac{1}{1 + \frac{4}{9} z^{-1}}$$

Ex.3

```
clear
clc
close all

p = [0.99 ; 0.99*exp(j*pi/8) ; 0.99*exp(-j*pi/8)];
z = [-1; exp(j*pi/16) ; exp(-j*pi/16)];

%a
figure, zplane(z,p)

%b
a = poly(p);
b = poly(z);

Ns = 1024;


%c % FREQUENCY IMPULSE RESPONSE
[H, w] = freqz(b,a,Ns);
 % w va da 0 a pi
 % column vectors
% WE WANT TO SEE ALSO THE NEGATIVE FREQUENCIES
% w and H are column vector (we have to flip the rows)
wc=[-flipud(w(2:end)); w];
Hc=[conj(flipud(H(2:end))); H];
% For flipping the columns we use the command fliplr

figure, subplot(2,1,1)
plot(wc,20*log10(abs(Hc)));
title('Module of the frequency response'),
xlabel('frequency [rad/sampl]'), ylabel('|H(\omega)|');
axis([-pi pi -50 50 ] )
```

```
subplot( 2 , 1 , 2 )
plot(wc, phase(Hc) )
title('Phase of the frequency response'),
xlabel('frequency [rad/sampl]'), ylabel('\angle(H(\omega))');
axis([ -pi pi -pi pi ])


% equivalently , the frequency response is computed as follows
%B = fft(b,Ns);
%A = fft(a,Ns);
%w = 2*pi*[0:Ns-1]/(Ns);
%H = B./A;
%figure
%subplot(2,1,1)
%plot(w, 10*log10(abs(H).^2));
%axis( [ 0 pi -50 50 ] )
%title('Module of the frequency response'),
%xlabel('frequency [rad/sampl]'), ylabel('|H(\omega)|');
%subplot( 2 , 1 , 2 )
%plot (w, phase (H) )
%axis ( [ 0 pi -pi pi ] )
%title('Phase of the frequency response'),
%xlabel('frequency [rad/sampl]'), ylabel('\angle(H(\omega))');


% impulse response

delta = [ 1 ; zeros(Ns-1,1)]';
h = filter(b,a,delta);
figure
subplot(2,1,1), stem([0:Ns-1],h), title('impulse response')

%h1=ifft(H);
%subplot(2,1,2), stem([0:Ns-1],h1), title('impulse response')



% filtering
% input
n=[0:1:Ns-1];
x= sin((pi/10)*n);
y=filter(b,a,x);
figure,
plot(abs(y))
```